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FERTILITY CHOICE, LAND, AND THE MALTHUSIAN HYPOTHESIS*

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1. INTRODUCTION

This paper shows that in a standard overlapping generations growth model (Diamond 1965), with a fixed amount of land and endogenous fertility, the competitive economy converges to a steady state with a zero population growth rate and positive consumption per capita. We interpret the Malthusian hypothesis as a positive statement about the relationship between population growth and consumption per-capita, when production exhibits diminishing returns to labor and there is a fixed amount of land essential for production. We show that, even when individuals care *only* about the number of their children and not about their children's welfare, the equilibrium is such that they eventually would choose to have only one child for each adult. Hence, if Malthus's "positive check" on population is the result of the response of optimizing agents to competitively determined prices, Malthus's pessimistic conjecture is not necessarily true, even though his other assumptions hold.

The choice of optimal population growth in Diamond's overlapping generations model was first considered by Samuelson (1975) (but, see Deardorff (1976)). Razin and Ben-Zion (1975) extended the model by assuming that fertility is subject to choice and that the utility of adults depends on both the number and welfare of their children. Nerlove, Razin, and Sadka (1985) analyzed Malthus's hypothesis in the same framework as Razin and Ben-Zion (1975), but assumed that land is fixed. They showed the competitive market is efficient in that world, but did not derive any positive conclusions.[1]

Niehans (1963) used a growth model with decreasing returns, and ad-hoc savings and fertility equations as in early neo-classical models. This paper addresses similar questions to those raised by Niehans (1963) within a general equilibrium, perfect foresight overlapping generations, model, where both savings and fertility are derived from the individual's choice problem. As in Niehans and Nerlove et al., diminishing returns to labor are due to the essentiality of a fixed amount of land. However, the rent on a unit of land, which is not discussed in Niehans and Nerlove et al., is determined endogenously in our model and is equal, in equilibrium, to the expected present value of its marginal product. We argue that our

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[1] The Nerlove, Razin, and Sadka (1986) model is similar to ours. It seems reasonable to predict that given our results, the positive implications of their model would also be similar. The formal derivation of the population growth rate in their infinite horizon optimization problem turns out to be technically much more difficult than in the Diamond model that we adopt. Note that given the fixed amount of land in our model, we have no reason to suspect that the allocation is not optimal (Kareken and Wallace 1977; McCallum 1986).

main result is due to the existence of a land market where land value depends on the future path of land per-capita.

It is the case here, as in most general equilibrium growth models, that multiple equilibria are not ruled out. However, the existence of (productive) land in our model implies that the model is *not* subject to an inefficient over-accumulation of capital as in Diamond (1965) (Tirole 1985 and McCallum 1986). Furthermore, as Calvo (1978) noted, the model has a unique steady state. We prove here that the unique steady state necessarily has a zero population growth and positive consumption per-capita.

In neoclassical growth models it is well known that, with no limits on production due to the existence of essential limited resources (Solow 1974) or when technical change is more powerful than population growth, the subsistence outcome can be avoided. We show that, given the endogeneity of population growth in neoclassical theory, neither existence of an essential factor nor absence of technical change would necessarily lead to "excessive" population growth. Finally, Solow (1974) and Mitra (1983) show that, with essential exhaustible resources and when population grows exponentially, a feasible positive consumption per capita program requires that population will eventually stabilize, i.e., zero population growth. Our result suggests that this requirement can be an outcome of the model if population growth is determined endogenously by the choice of individuals in the economy.[2]

In Section 2 of this paper, we present the model and discuss the characteristics of the economy when population is exogenously determined. We do so briefly because the results have been derived by others. Section 3 presents the case where population growth is endogenously determined, and Section 4 concludes.

2. THE MODEL

We consider a standard overlapping generations growth model (Diamond 1965) with land and endogenous population. The technology is represented by a constant returns to scale aggregate production function $F(K, L, R)$ where K is capital, L is labor, and R is land, such that $f(k, r) = F(K/L, 1, R/L)$ where $k = K/L$ and $r = R/L$. The single good can either be consumed or stored as capital for next period production. Capital depreciates at rate δ in storage and production. Land cannot be consumed directly and does not depreciate in production. Individuals live for three periods, as infants who make no decisions in the first period, as workers ("young") in the second period, and finally as retired ("old") in the third period. In the second period, individuals supply one unit of labor and decide upon life cycle consumption (savings) and the quantity of own children. Individuals are assumed to enjoy parenthood, and children are costly to bear and rear; each child born at time t consumes e units of the good.

[2] Mitra (1983) showed that there exists feasible programs with growing population when population growth is not exponential. Within our overlapping generations model, when fertility per-capita is a choice variable, population is growing exponentially. However, our model is somewhat less restrictive because land is not as limited as an exhaustible resource.

The representative individual of generation t has lifetime utility function[3]

$$U(C_1(t)) + \beta U(C_2(t)) + V(n(t + 1)) \tag{1}$$

where $C_i(t)$ is the consumption of a member of generation t at period $i + 1$ of the individual's life ($i = 1, 2$), and $n(t + 1)$ is the number of children (fertility) of each member of generation t.[4] The utility function satisfies the standard concavity and differentiability conditions with respect to all variables, and to assure positive consumption, it is assumed that $U'(0) = \infty$.

At time t the economy consists of $N(t + 1)$ infants, $N(t)$ young, and $N(t - 1)$ old. The economy begins at $t = 1$ with $N(0)$ old and $N(1)$ as initial conditions. Each of the initial old is endowed with $K(1)$ units of capital and $R/N(0)$ units of land, where R is the aggregate fixed stock of land. Since all individuals are assumed to be alike, there are $N(t) = n(t)N(t - 1)$ young at each period $t \geq 1$. Each of the old at time t owns $K(t)$ units of capital and $R(t) = R/N(t - 1)$ units of land. Since each young supplies one unit of labor, the number of workers at t is $N(t) = L(t)N(t - 1)$ with $L(t) = n(t)$ the number of workers per old at time t.

The Malthusian Result. Following the standard interpretation of Malthus with respect to production, we make three assumptions[5]: (a) the marginal product of labor is decreasing; (b) land is an essential factor of production and its quantity is fixed; and (c) population growth is exponential and cannot be directly reduced by individual choice. The above production function $F(K, L, R)$ is assumed to satisfy assumption (a). The conventional definition of essentiality in the literature (Solow 1974) implies that production converges to zero as land per worker approaches zero, for any positive level of capital per-capita. Assumption (c) implies that the sequence of the net population growth rates $[n(t)]_{t=0}^{\infty}$ is given *exogenously*, such that $n(t) > 1$ for all t, and the limit of the sequence, if it exists, is greater than one. Under assumptions (a) to (c), the model is consistent with Malthus's predictions; that is, consumption per-capita approaches or reaches zero (subsistence).[6] In fact, it does not matter whether the allocation is determined by competitive markets or a "social planner." Every allocation will switch eventually and lead to subsistence consumption.[7]

Malthus permitted population growth to change with consumption and defined subsistence consumption to be the level at which population was stable.

[3] As we mention in the introduction, we restrict the utility function so as not to include the utility of the children. The restriction considerably eases the mathematics, however.

[4] Alternatively one can view $n(t + 1)$ as the number of surviving children given a fixed and known child mortality rate, i.e. as the net fertility rate.

[5] See Malthus' (1798) discussion of his hypotheses on pages 70–71.

[6] The proof of this intuitive claim is straightforward from the per-capita economy budget constraint. Aggregate consumption per capita is equal to production per capita minus investment per capita. Given that $n(t) > 1$ for all t, and the essentiality of land, it can be shown that production per capita approaches zero or reaches zero in finite time. See our forthcoming paper.

[7] One can extend the model with exogenous fertility to make $n(t)$ a function of endogenously determined variables or some other exogenous checks on population (death rates). However, the results presented here would hold as long as $n(t)$ is not part of the individual choice set and/or $n(t) > 1$ for all t.

Although, as just noted, we have simplified the analysis by assuming that population growth is independent of consumption, it is easy to accommodate this notion of subsistence. To do so, define subsistence consumption to be a value, $\varepsilon > 0$, such that, for all levels of consumption per capita below ε, population is constant. Our result then is that the economy will reach this subsistence level of consumption in a finite time and will remain there as long as population is constant. Hence, as long as fertility is not a choice, and population is growing, essentiality of land implies the Malthusian result.

3. ENDOGENOUS FERTILITY IN A COMPETITIVE ECONOMY

In the competitive economy the problem of a young person at generation t, who is born at $t-1$, is to maximize (1) subject to:

$$C_1(t) = W(t) - K(t+1) - P(t)R(t+1) - en(t+1), \qquad e > 0 \tag{2}$$

$$C_2(t) = F(K(t+1), L(t+1), R(t+1)) - W(t+1)\,L(t+1) + (1-\delta)K(t+1) + P(t+1)R(t+1) \tag{3}$$

by choice of $K(t+1)$, $R(t+1)$, $n(t+1)$ and $L(t+1)$, where e is the consumption cost per child. Each of the young of generation t saves $K(t+1)$ units of the single consumption good for use in production at time $t+1$ and purchases $R(t+1)$ units of land for the same purpose at price per unit $P(t)$. Each supplies exactly one unit of labor, receives as a wage $W(t)$ units of the consumption good, and decides about his/her fertility level. At time $t+1$, each of the old of generation t hires $L(t+1)$ units of labor for production using the accumulated capital $K(t+1)$ and purchased land $R(t+1)$, and consumes the net of labor cost production, the non-depreciated quantity of capital, and the revenues from selling the non-depreciated land.

The first-order necessary conditions for a maximum are:

$$-U'(C_1(t)) + [F_K(K(t+1), L(t+1), R(t+1)) + (1-\delta)]\beta U'(C_2(t)) \leq 0 \quad \text{with} = \text{if } K(t+1) > 0 \tag{4}$$

$$[F_L(K(t+1), L(t+1), R(t+1)) - W(t+1)]\beta U'(C_1(t)) \leq 0 \quad \text{with} = \text{if } L(t+1) > 0 \tag{5}$$

$$-P(t)U'(C_1(t)) + [F_R(K(t+1), L(t+1), R(t+1)) + P(t+1)]\beta U'(C_2(t)) \leq 0 \quad \text{with} = \text{if } R(t+1) > 0 \tag{6}$$

$$-U'(C_1(t))e + V'(n(t+1)) \leq 0 \qquad \text{with} = \text{if } n(t+1) > 0. \tag{7}$$

In addition to the existence of non-negative values of $K(t+1)$, $L(t+1)$, $R(t+1)$, $n(t+1)$, $W(t)$ and $P(t)$, which satisfy (4) through (7), a perfect foresight competitive equilibrium requires that land and labor markets clear, i.e.,

$$L(t)N(t-1) = N(t) \tag{8}$$

and

$$R(t+1)N(t) = R. \tag{9}$$

Our assumptions so far ensure that (4), (5), and (6) hold as equalities. The rates of return on capital and land are equal, and the marginal product of labor is equal to the wage rate, as are standard results. Given the homogeneity of the population, each old individual in equilibrium employs $L(t+1) = n(t+1)$ workers. It is as if children work for their parents.

Equations (4) and (6) imply that

$$\frac{U'(C_1(t))}{\beta U'(C_2(t))} = F_K(t+1) + (1-\delta) = \frac{n(t+1)q(t+1) + F_R(t+1)[R/N(t)]}{q(t)} \tag{10}$$

where $F_K(t+1)$, $F_R(t+1)$ are the marginal products of K and R, respectively, at time $t+1$ and $q(t) = P(t)R(t+1) = P(t)[R/N(t)]$ is the value of land per-worker at time t.

Equation (7) determines the relationship between population growth and consumption. Due to the additive separability of the utility function and the fixed cost per child (e), independent, for example, of the number of other children, fertility and first period consumption move together.[8] If we wish to preserve the Malthusian result about eventual subsistence, we may assume that $V'(n) \to \infty$ as $n \to 1$ from above, and that $V(n) = -\infty$ for $n \leq 1$. (For example, $V(n) = \ln(n-1)$). With these preferences, the individual will choose $n(t) > 1$ for all t, and, as already explained, consumption will eventually equal zero. However, this assumption about preferences does not seem to be attractive because the essentiality of more than two children per family is not an obvious lower bound for individuals; it is also not an explicit assumption made by Malthus.

Without restricting preferences in this fashion, we can derive the following proposition:

PROPOSITION 1. *There exists a steady state with a positive constant per-capita consumption level if* and *only if $n(t)$ is equal to 1 for all $t > t_0$. This steady state is unique for given initial endowments.*

PROOF. To prove the necessary condition, assume the existence of a steady state with $C_{1t} = C_1$ and $C_{2t} = C_2$ for all $t > t_0$. Then, from (10), $F_K(t+1)$ is constant. This is the case only if $R/N(t+1)$ is constant due to the essentiality of land in production.[9] So, $R/N(t+1)$ is constant only if $n(t) = 1$ for all $t > t_0$. Note that this is consistent with Equation 7. If $n < 1$ then population decreases, and no steady state may exist.

[8] In Eckstein and Wolpin (1985) we show that if e includes time costs for raising children, then an increase in first period consumption would be associated with a decrease in the number of children.

[9] Due to the essentiality of land, $F_{KR} \neq 0$ and, as land per-capita changes over time, so does the marginal product of capital. Furthermore, from the Malthusian result above (footnote 6), there is no steady state with $C_1 > 0$, $C_2 > 0$ and $n > 1$.

The sufficient condition is derived from (7) where, if $n(t) = 1$ for $t > t_0$, then $C_1(t)$ is a constant for all $t > t_0$. The first equality in (10) implies that there exists a unique level of k, which determines C_2. (Here it is sufficient but not necessary to assume that $f(0, r) = 0$ and $f'(0, r) = \infty$). Q.E.D.

Let one plus the real rate of interest be equal to the marginal rate of substitution between consumption today and tomorrow. Then, $F_K(t + 1) + (1 - \delta) =$ one plus the real interest rate (see (10)), and by forward induction (see also Calvo 1978) and the boundness of $q(t)$ from above, (equation (2)), we get that the current value of land is equal to the present value of all future values of the product $F_R(t + 1)[R/N(t + 1)]$. Further, this result holds because, at the steady state, equation (10) guarantees that the real rate of interest is positive. This result follows from the fact that $q(t) = q > 0$ for all $t > t_0$, so that the right side of (10) implies that $F_K + 1 - \delta > n = 1$ at the steady state. Therefore, the standard arguments for inefficiency of the allocation do not hold here (see also Kareken and Wallace 1977, Tirole 1985, and McCallum 1986).

The model implies that the current value of land in equilibrium depends on the expected future path of land productivity. Land productivity is crucially affected by the population growth of the economy. Hence, existence of a perfect foresight equilibrium implies that agents can foresee an equilibrium path where the value of land is finite and positive at the steady state. Therefore, even though individuals today *do not* care about the utility and consumption of individuals in the future, they have to forecast the entire future path of fertility in evaluating the current value of land.

So far we have shown only that a unique steady state with $n = 1$ exists, and, if the competitive economy converges to a steady state, it also converges to a constant population. Is there a competitive economy that in fact satisfies this predicted convergence path? The answer is positive, and we prove it using an example.

An Example. We consider the Cobb-Douglas example where capital is fully depreciated in production ($\delta = 1$), where the utility function is logarithm additive so that equation (1) is given by:

$$\beta_1 \ln C_1(t) + \beta_2 \ln C_2(t) + \beta_3 \ln n(t + 1)$$

and where production is Cobb-Douglas

$$\begin{aligned} F(K(t + 1), L(t + 1), R(t + 1)) &= AK(t + 1)^{\alpha_1} L(t + 1)^{\alpha_2} R(t + 1)^{1 - \alpha_1 - \alpha_2} \\ &= AL(t + 1) k(t + 1)^{\alpha_1} r(t + 1)^{1 - \alpha_1 - \alpha_2} \end{aligned} \tag{11}$$

with $k(t) = K(t)/L(t)$ and $r(t) = R(t)/L(t)$.

These assumptions reduce the technical difficulty of the proof.[10] The reader should be aware that they represent an extreme case, however, that should work

[10] General characterizations of the equilibrium sequences of the model are technically very demanding. Only recently has Tirole (1985) provided a full characterization of Diamond's (1965) model. We do not see a large gain in repeating that exercise for the case of essential land and endogenous fertility. Furthermore, the example here shows the non-emptiness of Proposition 1.

against the intended result that $n_t = 1$ as $t \rightarrow \infty$. When $\delta = 1$ there is *less* gain from capital accumulation, and so there is greater likelihood of eventual subsistence. The Cobb-Douglas production function is the standard example for an economy with essential fixed inputs (Solow 1974 and Mitra 1983, among others). The logarithm additive utility function implies that savings are proportional to current income alone, as in the standard Solow growth model, and so also reduces the impact of the future on the current fertility decision.

Algebraic manipulations of the first-order conditions, the budget constraints and market clearing relationships yield the follow equations:

$$(12) \qquad n(t+1)k(t+1) + P(t)R(t+1) = \left(\frac{\beta_2}{\beta_1 + \beta_2 + \beta_3}\right) A\alpha_2 k(t)^{\alpha_1} r(t)^{1-\alpha_1-\alpha_2} = sW(t)$$

$$(13) \qquad n(t+1) = \frac{1}{e}\left(\frac{\beta_3}{\beta_1 + \beta_2 + \beta_3}\right) A\alpha_2 k(t)^{\alpha_1} r(t)^{1-\alpha_1-\alpha_2} = \frac{\beta_3}{\beta_2 e} sW(t)$$

where $s = \beta_2/(\beta_1 + \beta_2 + \beta_3)$ is the marginal rate of savings and $W(t)$ is the equilibrium wage rate.

The fertility rate is thus seen to be a constant fraction of first period income. Fertility, and thus population growth, is greater, the lower the cost of children and the greater their psychic benefit. Notice that these two equations contain three unknowns, $n(t+1)$, $k(t+1)$, and $P(t)$.[11] Substituting (13) into (12) yields

$$(14) \qquad \frac{\beta_3}{\beta_2 e} sW(t)k(t+1) + q(t) = sW(t)$$

where as before $q(t) = P(t)R/N(t)$.

An equilibrium for this economy consists of a time path for $[q(t), K(t+1), n(t+1)]_{t=1}^{\infty}$ that satisfies (12) and (13) and the initial conditions.[12] Suppose, as a possible solution, we conjecture that each individual divides his savings portfolio proportionally between land and capital, so that

$$(15) \qquad q(t) = \theta s W(t)$$

where θ is a constant proportion between zero and one. Then for a constant θ, the solution for $k(t)$ is[13]

$$(16) \qquad k(t) = (1-\theta)\frac{\beta_2}{\beta_3} e \qquad \text{for all } t \geq 2,$$

[11] Recall that $R(t+1) = R/N(t)$ and $N(t) = n(t)n(t-1)n(t-2)\cdots n(1)N(0)$.

[12] It is possible that there exists multiple equilibria, particularly since there is not an initial condition for the price of land. But there is only *one* steady state.

[13] Using the second equality in (10) it can be shown that there exists a unique $\theta > 0$ that satisfies $s\alpha_2\theta^2 + (1-\alpha_1 - a_2 s)\theta - (1-\alpha_1-\alpha_2) = 0$.

and for the population growth rate

$$n(t+1) = \frac{1}{e}\left(\frac{\beta_3}{\beta_1+\beta_2+\beta_3}\right)A\alpha_2\left[(1-\theta)\frac{\beta_2}{\beta_3}e\right]^{\alpha_1} r(t)^{1-\alpha_1-\alpha_2} \tag{17}$$
$$= Br(t)^{1-\alpha_1-\alpha_2}.$$

We have thus proved that there exists an equilibrium path for the economy, which is characterized by constant capital per capita.

Equation (17) can be written as

$$n(t+1) = B\left(\frac{R}{N(0)}\right)^{1-\alpha_1-\alpha_2}\left[\frac{1}{n(1)n(2)\cdots n(t)}\right]^{1-\alpha_1-\alpha_2}$$

and by recursive substitution of $n(t)$

$$n(t+1) = n(2)^{(\alpha_1+\alpha_2)^{t-2}}. \tag{18}$$

It is apparent that since $\alpha_1 + \alpha_2 < 1$, population growth, or the fertility rate, converges to unity. Thus, the competitive equilibrium is characterized by *zero* population growth in the steady state. If $n(2)$ is bigger than unity, then convergence is from above, while if $n(2)$ is less than unity, convergence is from below. Whether $n(2)$ is above or below unity depends upon the given level of $n(1)$ and the other parameter values. For example, the lower the cost of children (e) the higher will be the fertility rate at each point along the path. Hence, if the cost of children is initially low, then along the competitive equilibrium path, capital per capita is constant, population declines, and income per capita ($W(t)$) decreases. Since in the stationary equilibrium $n = 1$, consumption per capita has a positive finite steady state level. Thus, when fertility is subject to choice, there exists a competitive equilibrium which avoids the Malthusian outcome, and converges to the unique steady state of the economy.

4. CONCLUDING REMARKS

What is most remarkable about our result is not that fertility control undermines the usual Malthusian result, since effective external fertility control, say through government intervention in the form of forced sterilization, could obviously do so; but rather that the decentralized economy where individuals are selfish and short-lived can lead to a nonsubsistence steady state given individual fertility control.[14] We also have shown that exogenous fertility is necessary for the Malthusian outcome. With exogenous high fertility, a decentralized economy eventually vanishes possibly even in a finite time, although the path is likely to be Pareto optimal. No redistribution of resources between generations can prevent this outcome. It is also the case that allocation with endogenous fertility is efficient. Hence, whether we should be pessimistic or optimistic about prospects

[14] Assuming that the utility of a parent directly depends on the utility of his/her offspring, given the ability to control fertility, the results trivially follow from the assumptions.

for long run per capita consumption depends upon our assumptions about the course of technology and the way human fertility is determined.

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