

# Household Interaction and the Labor Supply of Married Women<sup>1</sup>

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## Abstract

Are changing social norms affecting the employment rates of women? A model is built in which the employment of husbands and wives is the outcome of potentially exogenously determined three different types of household games: The Classical household, where the spouses play a *Stackelberg* leader game in which the wife's labor supply decision is based on her husband's employment outcome. Second, The Modern household which characterized by a symmetric and simultaneous game solves as Nash equilibrium, and the Cooperative household where the couple jointly maximizes the weighted sum of their utilities. In all models husbands' employment is similar while wives work much less in Classical households.

Keywords: dynamic discrete choice, household labor supply, household game

JEL: E24, J2, J3

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## 1. Introduction

Cultural change (in the form of evolving social norms) can affect the employment patterns of married men and women by altering the interaction between spouses. Employment decisions are hypothesized to be an outcome of a game played in the household, which is assumed to be one of three alternative types. We assume that social norms exogenously determine the type of game played in the household and hypothesize that certain types of games encourage higher female employment than others.

The goal of this paper is to empirically measure the change in female employment due to a shift from one household game to another. To do so, we assume that there are three alternative types of households: “Classical” (C), “Modern”(M) and "Cooperative"(COP). In the Classical household, the husband is a “Stackelberg leader”. In other words, he is the first to make an employment decision in each period, taking into account the best forecast of his wife’s employment outcome. The wife treats her husband’s decision as exogenously given. The use of the Stackelberg game is a natural extension of models in the early literature, which focused primarily on only one spouse, and where the decision of the other spouse was taken as given (i.e. exogenous). Becker (1973) argues that the division of labor in the household results in the wage of each spouse being a substitute for the wage of the other. Thus, for example, if one spouse has a lower wage than the other, it will be less costly for him/her to stay at home and therefore in general women are more likely to choose to stay at home. A woman will search for work only if her husband’s income is below some threshold. Becker’s model is consistent with our Classical household.<sup>4</sup>

In both the Modern and Cooperative households, the male and the female are equal players who act simultaneously and possess symmetric information. The main difference between them is that in the Modern household the couple plays a non-cooperative game that might result in an inefficient equilibrium, while in the cooperative household they cooperate in order to maximize a joint utility

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<sup>4</sup> This framework appears in the empirical literature on female employment. See Heckman and MaCurdy (1980,1982), Hotz and Miller (1988), Eckstein and Wolpin (1989) and Van Der Klaauw(1996), among others.

function and therefore the solution is efficient.<sup>5</sup> The characteristics of the Cooperative household are based on the collective household framework developed by Chiappori (1988, 1992, 2002).<sup>6</sup> The couple in this type of household maximizes the weighted sum of their individual utilities. The weights are determined according to the bargaining power of each partner. The Modern household follows a Nash game and its outcomes are not necessarily efficient.<sup>7</sup>

In order to empirically estimate the impact of different social norms on male and female employment, we assume that married couples can be divided into three types. The type of household is unobserved and is determined exogenously with a given probability. This probability is specified as a function of the couple's ages, education and level of assortative mating, as measured by the gap in age and education. As a result, we are able to estimate the effect of a change in the proportion of each type of household, which we attribute to changing social norms, on the female employment rate.

The model is characterized by three endogenous labor market states: employment, unemployment and out of the labor force. Wage offers are given exogenously as a random outcome that follows a logit probability function and wage levels follow the standard Mincer/Ben-Porath wage equation. Households are characterized by a common budget constraint and joint consumption where divorce is a potential exogenous event that occurs randomly, conditional on the household state. Children are added randomly depending on the state of the household and parents' utility increases with the quality and quantity of their children. We restrict the model such that preferences and market opportunities are the same for the male and female in all types of households. Under these conditions, we find that females in

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<sup>5</sup> Bresnahan and Reiss (1991) formulated a method for estimating discrete games using three alternative specifications (a simultaneous-move game, a sequential-move game and a cooperative specification) which are equivalent to the three types of households considered here.

<sup>6</sup> Recent empirical papers, such as Mazzocco, Ruiz and Yamaguchi (2007), Jacquemet and Robin (2009), Fernández and Wong (2011) and Gemici and Laufer (2011), use Chiappori's model.

<sup>7</sup> Del Boca and Flinn (2010), Brown and Flinn (2007) and Tartari (2005) also use a non-cooperative household game.

the Modern household are predicted to have higher employment rates than females in the Classical household if one of the following conditions holds: (i) women earn less than men; (ii) the risk aversion parameter is lower for women than for men (i.e., women have higher relative risk aversion). This is because the employment decisions of women in the Classical household are made given the realized outcome of the husband. Women in both the Modern and Cooperative households know the expected outcome of the husband, though not its realization, and therefore we expect them to have similar employment rates. Nevertheless, since women in Cooperative households maximize the weighted sum of their utility and that of their husbands, the marginal gain from employment is higher due to the impact of one's own wage on the well-being of one's spouse.

We estimate the model using the Simulated Moments Method (SMM) and a PSID sample of 863 couples who married in 1983-4, for whom there is up to ten years of quarterly data. In order to focus on internal family interactions, we assume that all the parameters are the same for the three types of households. The estimated model provides a good fit to the trends in employment, unemployment, wages and other moments of household labor supply and the estimated parameters are consistent with the theory and results presented in the literature. Thus, the estimated employment rate of women in Modern (Cooperative) households exceeds that of women in Classical households by 12 (16) percent, while the employment rate of men in each type of household is about the same. Since men have higher job-offer rates and higher potential wages, they have broader employment choices in all types of households. However, given the simultaneous choices in Modern and Cooperative households and the higher level of risk aversion among women, more women in Modern and Cooperative households choose to participate in the workforce and they also work more than their counterparts in Classical households.

The rest of the paper is organized as follows: Section 2 presents a dynamic household labor supply model and also presents a detailed solution of the model for the three types of households. Section 3 describes the PSID data and the estimation method. Section 4 presents the estimation results and the fit to the data. Section 5 discusses counterfactuals of the model and Section 6 concludes.

## 2. The Household Model

We consider a household of two players, wife and husband, who are indexed by  $j = \{W, H\}$ . From the point in time at which a couple marries ( $t = 0$ ), their household is categorized as playing one of three games, which will be specified below. We first specify the common features of the games and then some of their more specific aspects. Following that, we specify the solution of each game's equilibrium. At the end of each period  $t$ , there are three outcomes denoted by  $a_{ij}^i$  such that  $a_{ij}^1 = 1$  if the individual is employed ( $E$ ),  $a_{ij}^2 = 1$  if the individual is unemployed ( $UE$ ) and  $a_{ij}^3 = 1$  if the individual is out of the labor force ( $OLF$ ).  $\sum_{i=1}^3 a_{ij}^i = 1$  and the labor market state for individual  $j$  is given by  $A_{ij} = \{a_{ij}^1, a_{ij}^2, a_{ij}^3\}$  and for both spouses by  $A_t = \{A_{tW}, A_{tH}\}$ .

We assume that each period  $t$ , from the wedding day ( $t = 0$ ) until retirement ( $t = T$ ), is divided into two sub-periods: during the first sub-period, an individual who is out of the labor force ( $a_{t-1j}^3 = 1$ ) or unemployed ( $a_{t-1j}^2 = 1$ ) decides whether or not to search for a job. If s/he chooses to search, i.e.  $d_{ij} = 1$ , then in the second sub-period, s/he receives at most one job offer and if s/he decides to accept it then  $a_{ij}^1 = 1$ . If s/he does not accept it or does not receive an offer, she is unemployed ( $a_{ij}^2 = 1$ ). If s/he does not search for a job, i.e.  $d_{ij} = 0$ , then in the second sub-period s/he is out of the labor force ( $a_{t-1j}^3 = 1$ ).

If s/he is initially employed ( $a_{t-1j}^1 = 1$ ), she may choose to quit, i.e.  $d_{ij} = 0$ , and then s/he is out of the labor force. Alternatively, if s/he does decide to search, i.e.  $d_{ij} = 1$ , then s/he may receive an offer. In the second sub-period, if s/he does not receive an offer, then this is equivalent to being fired and

therefore s/he becomes unemployed. If s/he does receive an offer, s/he either decides to accept it ( $a_{ij}^1 = 1$ ) and stay employed or reject it and become unemployed.<sup>8</sup>

Consumption ( $x$ ) is a joint family outcome and as a result the household budget constraint in each period  $t$ ,  $t=1, \dots, T$  is given by:

$$(2.1) \quad x_t = (y_{iW} \cdot a_{iW}^1 + y_{iH} \cdot a_{iH}^1 + b_{iW} \cdot (1 - a_{iW}^1) + b_{iH} \cdot (1 - a_{iH}^1)) \cdot (1 - \theta(N_t))$$

where  $y_{iW}$  and  $y_{iH}$  are the wages,  $b_{iW}$  and  $b_{iH}$  are the unemployment benefits and  $x_t$  is the couple's joint consumption during period  $t$ . For simplicity, we define the cost per child (per-child consumption) in goods where  $\theta$  is a given proportion of family income devoted to the children.<sup>9</sup>  $N_t$  is the number of children in the household, which is given by  $N_t = N_{t-1} + n_t$ , where the event of birth,  $n_t = 1$ , is a given random event that depends on employment and other states of the household.

We adopt the Mincerian/Ben-Porath wage function for each  $j = \{W, H\}$  where experience is endogenously determined, such that:

$$(2.2) \quad \ln y_{ij} = \beta_1^j + \beta_2^j K_{t-1j} + \beta_3^j K_{t-1j}^2 + \beta_4^j S_j + \varepsilon_{ij}^1.$$

where  $K_{t-1j}$  is actual work experience accumulated by the individual according to  $K_{ij} = K_{t-1j} + a_{ij}^1$ , for which the initial value is the level of experience on the day of the wedding and  $S_j$  denotes the individual's predetermined years of schooling.  $\varepsilon_{ij}^1$  is the standard normally distributed zero-mean, finite-variance and serially independent error, which is uncorrelated with  $K$  and  $S$ .

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<sup>8</sup> If a person is employed in two or more sequential periods, we are unable to determine whether he switched jobs or stayed in the same job. Therefore, "on the job search" is characterized by a new wage offer, although we assume that search costs are zero when employed.

<sup>9</sup>  $\theta = 0.2$  if  $N = 1$ ,  $\theta = 0.32$  if  $N = 2$ ,  $\theta = 0.41$  if  $N = 3$ ,  $\theta = 0.48$  if  $N > 3$  - following the OECD equivalent scale.

Utility from consumption is characterized by constant relative risk aversion and utility from leisure and children is linear, such that: <sup>10</sup>

$$(2.3) \quad U_{ij} = u_j(x_t) + \alpha_{1j} \cdot l_{ij} + \alpha_{2j} \cdot f(l_{iW}, l_{iH}, x_t, N_t),$$

where  $u_j(x_t) = (x_t)^{\gamma_j} / \gamma_j$  is utility from total household consumption,  $l_{ij}$  is the individual's leisure and  $f(l_{iW}, l_{iH}, x_t, N_t)$  is a CES function determining the utility the couple derives from the quantity and quality of children:

$$(2.4) \quad f(l_{iW}, l_{iH}, x_t, N_t) = (\rho_1 + \rho_{11} \cdot INF_t) \cdot (l_{iW})^{\rho_0} + (\rho_2 + \rho_{22} \cdot INF_t) \cdot (l_{iH})^{\rho_0} + \rho_3 \cdot \left(\frac{\theta}{1-\theta} \cdot x_t\right)^{\rho_0} + (1 - \rho_1 - \rho_{11} \cdot INF - \rho_2 - \rho_{22} \cdot INF - \rho_3) \cdot N_t^{\rho_0} \Big)^{\rho_0}$$

Children's utility is a function of consumption per child, the number of children and parents' leisure, where the utility from leisure is higher if there is an infant aged 1-3 ( $INF=1$ ). By inserting the budget constraints (equation (2.1)) into the current utility (equation (2.3)), we obtain the wife's utility for each employment state:

$$(2.5) \quad \begin{aligned} U_{iW}^E &= u_W \left( (1-\theta)(y_{iW} + y_{iH} \cdot a_{iH}^1) \right) + \alpha_{1W} \cdot f(l_{iW}, l_{iH}, x_t, N_t) \\ U_{iW}^{UE} &= u_W \left( (1-\theta)(y_{iH} \cdot a_{iH}^1) \right) + \alpha_{1W} \cdot f(l_{iW}, l_{iH}, x_t, N_t) + \alpha_{2W} \cdot (l_{iW} - SC_j) + \varepsilon_{iW}^2 \\ U_{iW}^{OLF} &= u_W \left( (1-\theta)(y_{iH} \cdot a_{iH}^1) \right) + \alpha_{1W} \cdot f(l_{iW}, l_{iH}, x_t, N_t) + \alpha_{2W} \cdot l_{iW} + \varepsilon_{iW}^3 \end{aligned}$$

When the wife is unemployed ( $a_{i-1W}^2 = 1$ ) the utility from leisure,  $\alpha_{1W} \cdot l_{iW}$ , is adjusted for the cost of search  $SC_j$  and  $\varepsilon_{iW}^2, \varepsilon_{iW}^3$  are utility shocks for the states of  $UE$  and  $OLF$ , respectively. The random shocks to preferences and wages are determined by the vector  $\varepsilon_{ij} = [\varepsilon_{ij}^1, \varepsilon_{ij}^2, \varepsilon_{ij}^3]$  which is assumed to be joint normal and serially uncorrelated, where  $\varepsilon_{ij} \sim N(0, \Sigma_j)$ , *i.i.d.* and  $\Sigma$  is unrestricted. Equivalently, the husband's utility for each employment state is given by:

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<sup>10</sup> We use the assumption that all earnings are consumed, i.e. neither saving nor borrowing is feasible, even though utility is not assumed to be linear. This assumption is extreme though standard in the modeling of dynamic labor supply.

$$\begin{aligned}
(2.6) \quad U_{iH}^E &= u_H \left( (1-\theta)(y_{iH} + y_{iW} \cdot a_{iW}^1) \right) + \alpha_{1H} \cdot f(l_{iW}, l_{iH}, x_t, N_t) \\
U_{iH}^{UE} &= u_H \left( (1-\theta)(y_{iW} \cdot a_{iW}^1) \right) + \alpha_{1H} \cdot f(l_{iW}, l_{iH}, x_t, N_t) + \alpha_{2H} \cdot (l_{iH} - SC_j) + \varepsilon_{iH}^2 \\
U_{iH}^{OLF} &= u_H \left( (1-\theta)(y_{iW} \cdot a_{iW}^1) \right) + \alpha_{1H} \cdot f(l_{iW}, l_{iH}, x_t, N_t) + \alpha_{2H} \cdot l_{iH} + \varepsilon_{iH}^3.
\end{aligned}$$

The individual can always choose to stay at home, i.e. OLF ( $a_{t-1j}^3 = 1$ ), even though there are other states available to him in each period  $t$ . Thus, the individual receives at most one job offer per period with its probability depending on the labor market common knowledge state variables. We use the following specification for this probability:

$$(2.7) \quad \Pr(\lambda_{ij}) = \frac{\exp(\rho_{01j} \cdot a_{t-1j}^1 + \rho_{02j} \cdot a_{t-1j}^2 + \rho_{03j} \cdot a_{t-1j}^3 + \rho_{1j} \cdot S_j + \rho_{2j} \cdot K_{t-1j})}{1 + \exp(\rho_{01j} \cdot a_{t-1j}^1 + \rho_{02j} \cdot a_{t-1j}^2 + \rho_{03j} \cdot a_{t-1j}^3 + \rho_{1j} \cdot S_j + \rho_{2j} \cdot K_{t-1j})},$$

where  $\lambda_{ij} = 1$  if individual  $j$  has a job offer in period  $t$ . We assume that in each period the individual may lose his job with a probability that is negatively correlated with his accumulated experience and education and depends on the time trend. The probability function for being laid off is identical to (2.7) except that it has different parameters values.

We supplement the model with several given dynamic probabilities for demographic characteristics, whose expectations are potentially important in determining household labor supply. The probability of having another child is a function of the woman's employment state in the previous period, the woman's age and education and those of her husband, the current number of children and the existence of an infant (*INF*). The probability of having an additional child is given by (as in Van der Klaauw, 1996):

$$(2.8) \quad \Pr(N_t = N_{t-1} + 1) = \Phi \left( \lambda_1 \cdot AGE_t^W + \lambda_2 \cdot (AGE_t^W)^2 + \lambda_3 \cdot AGE_t^H + \lambda_4 \cdot S^W + \lambda_5 \cdot S^H + \lambda_6 a_{t-1W}^1 + \lambda_7 a_{t-1H}^1 + \lambda_8 \cdot N_t + \lambda_9 \cdot INF_t \right)$$

where  $\Phi(\cdot)$  is the standard normal distribution function. The probability of divorce is estimated as a function of how long the couple has been married ( $t$ ), the current number of children and the partner's education:

$$(2.9) \quad \Pr(M_t = 0 / M_{t-1} = 1) = \Phi \left( \xi_1 \cdot t + \xi_2 \cdot t^2 + \xi_3 \cdot N_t + \xi_4 \cdot S^W + \xi_5 \cdot S^H \right)$$

In the terminal period  $T$ , we use a linear approximation of the value function in the final period:



$$(2.10) \quad U_{Tj} = \delta_{1j} + \delta_{2j} \cdot K_{T-1W} + \delta_{3j} \cdot K_{T-1H} + \delta_{4j} \cdot N_{T-1} + \delta_{5j} \cdot INF_{T-1} + \delta_{6j} \cdot a_{T-1W}^1 + \delta_{7j} \cdot a_{T-1H}^1$$

The state space for each period  $t$  and for both sub-periods is commonly observed by both husband and wife and is given by  $\Omega_t = [K_{t-1H}, K_{t-1W}, S_H, S_W, A_{t-1H}, A_{t-1W}, N_t, INF_t]$ . Let  $U_y(A_t, \Omega_t, \varepsilon_t)$  be the per-period utility of household member  $j$  as explicitly formulated by equations (2.5) and (2.6), where  $\varepsilon_t = [\varepsilon_{tW}, \varepsilon_{tH}]$ . The expected lifetime utility of each member of the household is given by

$$E[\sum_{t=0}^T \beta^t U_y(A_t, \Omega_t, \varepsilon_t) | \Omega_0].$$

We consider three alternative games within the household that determine the labor supply outcome. The first is referred to as "Classical" (C) where the husband plays the role of a Stakelberg leader, such that in each period he moves first and the wife responds after the husband's outcomes are revealed. The second game is referred to as "Modern" (M), since it involves the simultaneous moves of both husband and wife before they know the spouse's outcomes.<sup>11</sup> The third game is referred to as "Cooperative" (COP) since the couple solve a collective utility function, as in Chiappori (1992). The strategies and the Markov Perfect Equilibrium (MPE) are formally described below.

In order to focus on the impact of the internal family game on household labor supply, we assume that utility functions, wage functions and job-offer rate parameters differ between husband and wife but are identical for all games within the household. Below we specify the structure of each game.

### ***2.1 The Classical Household Labor Supply***

The main assumption in this type of household is that the wife makes her decisions after the outcomes of her husband are realized. The solution of the Classical household game within each period  $t$  is divided into two sub-periods and is solved backwards from the end of the period. We solve for the

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<sup>11</sup> As mentioned, Del Boca and Flinn (2010) specify the intra-household game to be endogenous where the alternatives are a cooperative or (inefficient) Nash equilibrium.

expected utility of the wife from search for each outcome of the husband. The husband chooses whether to search conditional on his wife's expected reaction to his potential outcome. Once the outcome of the husband is known, the wife chooses whether or not to search and then, if she receives an offer, she chooses whether to be employed.

Let  $V_{ij}^{d_{ij}}(A_{t-j}, \Omega_t, e_{ij})$  be the value function of player  $j$  for a strategy  $d_{ij}$ , which is a function of his partner's outcome,  $A_{t-j}$ ,  $\Omega_t$  and  $e_{ij}$ . Here  $e_{ij}$  is the expected values of  $\varepsilon_i$  which are known to  $j$  in the sub-period when  $j$ 's decisions are made. The formal solution is obtained in 5 steps as follows: In step 1, we find the husband's expected values for his wife's decisions. To do so, we solve for the wife's best response function to each of her husband's outcomes. The wife's decision is whether or not to search. Her value function from search in the first sub-period is defined by:<sup>12</sup>

$$(2.11) \quad V_{iW}^1(A_{iH}, \Omega_t, e_{iH}) = \{\Pr(\lambda_{iW}) \cdot \max[v_{iW}^E(A_{iH}, \Omega_t, e_{iH}), v_{iW}^{UE}(A_{iH}, \Omega_t, e_{iH})] + (1 - \Pr(\lambda_{iW})) \cdot v_{iW}^{UE}(A_{iH}, \Omega_t, e_{iH})\}$$

The wife's value function from choosing not to search is defined by:

$$(2.12) \quad V_{iW}^0(A_{iH}, \Omega_t, e_{iH}) = v_{iW}^{OLF}(A_{iH}, \Omega_t, e_{iH})$$

where  $\{\Pr(\lambda_{iW})\}$  is the job offer probability (equation 2.7) and  $v_{iW}^E(A_{iH}, \Omega_t, e_{iH})$ ,  $v_{iW}^{UE}(A_{iH}, \Omega_t, e_{iH})$ ,  $v_{iW}^{OLF}(A_{iH}, \Omega_t, e_{iH})$  are the value functions of  $E$ ,  $UE$  and  $OLF$ , given the husband's information. The definition of these functions is fully described in the Appendix (A). The Bellman equation of the wife's DP problem is given by:

$$(2.13) \quad V_{iW}^t(A_{iH}, \Omega_t, e_{iH}) = \max_{d_{iW}} \{V_{iW}^{d_{iW}}(A_{iH}, \Omega_t, e_{iH})\}.$$

Since the solution of the DP problem is a function of the husband's outcome,  $A_{iH}$ , we evaluate (2.13) for each of his outcomes. The wife's best response function is the strategy that maximizes her expected utility for each of her husband's outcomes and is given by:

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<sup>12</sup> Since we calculate the husband's expected values for his wife's decision, the wife's value functions depend on the husband's information regarding the error term,  $e_{iH}$ .

$$(2.14) \quad b_{iW}(A_{iH}, \Omega_t, e_{iH}) = \arg \max_{d_{iW}} \{V_{iW}^{d_{iW}}(A_{iH}, \Omega_t, e_{iH})\},$$

In step 2, given the above best response function of the wife, i.e. (2.14), we can solve for the husband's value from search, i.e.  $V_{iH}^1(A_{iW}^e, \Omega_t, e_{iH})$  and from no search, i.e.  $V_{iH}^0(A_{iW}^e, \Omega_t, e_{iH})$ . Here,  $A_{iW}^e$  is the wife's expected outcome, given the best response function that defines the wife's optimal strategy for any  $A_{iH}$ . We can now choose the strategy  $d_{iH} = \{0,1\}$  that maximizes:

$$(2.15) \quad V_{iH}^t(A_{iW}^e, \Omega_t, e_{iH}) = \max_{d_{iH}} \{V_{iH}^{d_{iH}}(A_{iW}^e, \Omega_t, e_{iH})\},$$

The husband's best response function is the strategy that maximizes his expected utility for each of his wife's expected outcomes:

$$(2.16) \quad b_{iH}(A_{iW}^e, \Omega_t, e_{iH}) = \arg \max_{d_{iH}} \{V_{iH}^{d_{iH}}(A_{iW}^e, \Omega_t, e_{iH})\},$$

If  $d_{iH}^1 = 0$ , then  $\alpha_{iH}^3 = 1$  and we proceed to step 4 (i.e. the wife's decision). If  $d_{iH}^1 = 1$ , we proceed to step 3. In step 3, we randomly draw whether the husband receives a job offer. If he does, he either will accept it and be employed if  $v_{iH}^E(A_{iW}^e, \Omega_t, e_{iH}) > v_{iH}^{UE}(A_{iW}^e, \Omega_t, e_{iH})$  or reject it and be unemployed. If he doesn't receive an offer, he will be unemployed.

In step 4, we solve the wife's employment decision. She already has information regarding her husband's decisions and outcome. Therefore, we calculate  $b_{iW}(A_{iH}, \Omega_t, e_{iW})$ , and solve for the strategy  $d_{iW} = \{0,1\}$  that maximizes  $V_{iW}^t(A_{iH}, \Omega_t, e_{iW})$ . In step 5, if her best response is  $d_{iW} = 0$ , then  $\alpha_{iW}^3 = 1$  and the game is solved. If her best response is  $d_{iW} = 1$ , we randomly draw whether she receives a job offer using equation (2.7). If she does not, then she is unemployed and the game is solved. If she does, then she will accept it and be employed if  $v_{iW}^E(A_{iH}, \Omega_t, e_{iW}) > v_{iW}^{UE}(A_{iH}, \Omega_t, e_{iW})$ .

The implications of the game for employment are discussed in section 2.4. The model is solved backwards from T as a Markov Perfect Equilibrium where steps 1-5 are repeated for each t. Since the Classical game is a fully recursive system, its solution follows that of a Stackelberg leader game and

therefore there always exists a unique sub-game perfect equilibrium (Bresnahan and Reiss, 1991).

The detailed solution for this game can be found at the Appendix (A).

## 2.2 The Modern Household Labor Supply

In a Modern household, the husband and wife make their decisions simultaneously and with symmetric information. Each maximizes his/her own expected utility for each of his/her partner's potential choices using the model's specified randomness. This game has two steps: in the first step, the husband and wife choose whether or not to search. They act simultaneously and have the same information: both observe  $\Omega_t$  and the realization of the out-of-labor-force shock,  $\varepsilon_{iH}^3, \varepsilon_{iW}^3$ , but they do not know the other two shocks,  $\varepsilon_{ij}^1, \varepsilon_{ij}^2$ , which will only be revealed in step 2. The utility of each state depends on the strategies of both the individual and his spouse. Therefore, we calculate the utility for all 2X2 (search or no-search) choices. If the husband does not search, the wife's utility from search is given by:<sup>13</sup>

$$(2.17) \quad V_{iW}^1(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 0) = \left\{ \Pr(\lambda_{iW}) \cdot \max [v_{iW}^E(\{0,0,1\}, \Omega_t, e_{iW}), v_{iW}^{UE}(\{0,0,1\}, \Omega_t, e_{iW})] + (1 - \Pr(\lambda_{iW})) \cdot v_{iW}^{UE}(\{0,0,1\}, \Omega_t, e_{iW}) \right\}$$

If the husband searches, the wife's utility from search is given by:<sup>14</sup>

$$(2.18) \quad V_{iW}^1(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 1) = \left\{ \begin{array}{l} \Pr(\lambda_{iH}) \cdot \Pr(\lambda_{iW}) \cdot \max [v_{iW}^E(\{1,0,0\}, \Omega_t, e_{iW}), v_{iW}^{UE}(\{1,0,0\}, \Omega_t, e_{iW})] + \\ \Pr(\lambda_{iH}) \cdot (1 - \Pr(\lambda_{iW})) \cdot v_{iW}^{UE}(\{1,0,0\}, \Omega_t, e_{iW}) + \\ (1 - \Pr(\lambda_{iH})) \cdot \Pr(\lambda_{iW}) \cdot \max [v_{iW}^E(\{0,1,0\}, \Omega_t, e_{iW}), v_{iW}^{UE}(\{0,1,0\}, \Omega_t, e_{iW})] + \\ (1 - \Pr(\lambda_{iH})) \cdot (1 - \Pr(\lambda_{iW})) \cdot v_{iW}^{UE}(\{0,1,0\}, \Omega_t, e_{iW}) \end{array} \right\}$$

<sup>13</sup> The value functions of the Modern household depend on the spouse's strategy but not the spouse's outcome, since the game is a simultaneous move game.

<sup>14</sup> For simplicity, we write the equation assuming that if the husband receives a job offer he will accept it. This is not always the case since he will accept the offer only if  $v_{iH}^E(A_{iW}, \Omega_t, e_{iH}) > v_{iH}^{UE}(A_{iW}, \Omega_t, e_{iH})$  as described in detail in B.

If the husband does not search, the wife's utility from not searching is:

$$(2.19) \quad V_{iW}^0(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 0) = v_{iW}^{OLF}(\{0,0,1\}, \Omega_t, e_{iW})$$

If the husband searches, the wife's utility from not searching is:

$$(2.20) \quad V_{iW}^0(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 1) = \Pr(\lambda_{iH}) \cdot v_{iW}^{OLF}(\{1,0,0\}, \Omega_t, e_{iW}) + (1 - \Pr(\lambda_{iH})) \cdot v_{iW}^{OLF}(\{0,1,0\}, \Omega_t, e_{iW})$$

The value function of the husband  $V_{iH}^1(d_{iW}, \Omega_t, e_{iH}), V_{iH}^0(d_{iW}, \Omega_t, e_{iH})$  has an equivalent structure.

Let  $V_{iW}^i(d_{iH}, \Omega_t, e_{iW})$  be the value function of the wife's DP problem. The Bellman equation is given by:

$$(2.21) \quad V_{iW}^i(d_{iH}, \Omega_t, e_{iW}) = \max_{d_{iW}} \{V_{iW}^{d_{iW}}(d_{iH}, \Omega_t, e_{iW})\},$$

The wife's best response function is given by:

$$(2.22) \quad b_{iW}(d_{iH}, \Omega_t, e_{iW}) = \arg \max_{d_{iW}} \{V_{iW}^{d_{iW}}(d_{iH}, \Omega_t, e_{iW})\}.$$

This best response function provides the wife's optimal strategy if the husband, both now and in the future, behaves according to his respective strategy  $d_{iH}$ . The husband's Bellman equation  $V_{iH}^i(d_{iW}, \Omega_t, e_{iH})$  and best response function  $b_{iH}(d_{iW}, \Omega_t, e_{iH})$  have an equivalent and symmetric form. A Markov Perfect Equilibrium (MPE) in this game is a set of strategies  $d_{iH}^*, d_{iW}^*$  such that for both players we have that  $d_{iW}^* = b_{iW}(d_{iH}^*, \Omega_t, e_{iW})$  and  $d_{iH}^* = b_{iH}(d_{iW}^*, \Omega_t, e_{iH})$ . In step 2, given the equilibrium  $d_{iH}^*, d_{iW}^*$ , we solve the couple's employment problem. If one of the couple chooses not to search, then  $\alpha_{ij}^3 = 1$  and if s/he does decide to search, then s/he receives a job offer with the probability given by equation (2.7). If one or both of them receives an offer, s/he chooses whether or not to accept it. If s/he does not receive a job offer, then s/he is unemployed, i.e.  $\alpha_{ij}^2 = 1$ . This second step is fully described and solved in the Appendix (B).

The Modern household game is solved as a standard Nash equilibrium. In other words, the values of the two choices for each of the family members are calculated in order to form a 2X2 matrix, which is used to determine a standard Nash equilibrium. As is well-known, a unique Nash equilibrium does not always exist. Therefore, we need to define the algorithm for both a game with no equilibrium and a game with multiple equilibria. In the case that there are more than one equilibrium for household  $i$  at period  $t$ , we need to specify an equilibrium selection mechanism<sup>15</sup>. In each iteration, with multiple equilibria, we compute the household's total welfare (sum of partner's utilities) for each equilibrium and restrict the algorithm to always solve for the equilibrium with the highest welfare. Given that this is a household problem, it seems reasonable that the household does not play a Pareto inferior equilibrium. If during the estimation it is found that no equilibrium exists, then the iteration will be stopped and a new set of parameters will be used. Therefore, in the case of the estimated parameters, there is a unique selected equilibrium for every household in each period.<sup>16</sup>

### ***2.3 The Cooperative Household Labor Supply***

The Cooperative household is based on Chiappori (1992), which is one of the leading models of household behaviour, and therefore a natural alternative to the Classical and Modern games. The couple maximizes a joint utility function which is generally considered to be a single player game against nature. It is solved backwards from period  $T$ , as a standard dynamic programming problem with a unique solution. As such, the Cooperative specification does not lead to a game between the partners.

The Cooperative couple acts simultaneously and with symmetric information, where they maximize a joint household utility function (unlike in the case of the Modern and Classical households), which is a weighted sum of both individuals' utility functions. Their utility is weighted according to the relative bargaining power ( $BP$ ) of each spouse.<sup>17</sup>

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<sup>15</sup> We follow a referee's suggestion to use the criteria of highest welfare for the household.

<sup>16</sup> It should be noted that for the set of estimated parameters we do not find multiple equilibria.

<sup>17</sup> In the estimation, we use  $BP=0.5$ . We also consider the case of  $BP=1$  and  $BP=0$ . See chapter 5.

$$(2.23) \quad V_t^{d_w d_H}(\Omega_t, e_{iW}, e_{iH}) = BP \cdot V_{iW}^{d_w}(d_{iH}, \Omega_t, e_{iW}) + (1 - BP) \cdot V_{iH}^{d_H}(d_{iW}, \Omega_t, e_{iH}) | d_{iW}$$

$V_t^{d_w d_H}(\Omega_t, e_{iW}, e_{iH})$  can have four possible values according to the search strategy of each spouse.

The optimization is solved in two steps: In step 1, we maximize (2.23). We define the solution at this step as a set of decisions,  $d_{iH}^*, d_{iW}^*$ , that solves the following function:

$$(2.24) \quad \max_{d_{iH}, d_{iW}} \{V_t^{01}(\Omega_t, e_{iW}, e_{iH}), V_t^{11}(\Omega_t, e_{iW}, e_{iH}), V_t^{00}(\Omega_t, e_{iW}, e_{iH}), V_t^{10}(\Omega_t, e_{iW}, e_{iH})\}$$

In step 2, we calculate the outcome of the search, given the decisions  $d_{iH}^*, d_{iW}^*$ . If one of the spouses chooses not to search, then  $\alpha_{ij}^3 = 1$ . Alternatively, if s/he does decide to search, then s/he receives a job offer with a probability given by (2.7). If one or both of them receives an offer, s/he chooses whether or not to accept it. If s/he does not receive a job offer, then s/he is unemployed, i.e.  $\alpha_{ij}^2 = 1$ . This game is fully described and solved in the Appendix (C).

#### ***2.4 Does Household Type Affect Female Employment?***

The aim of the paper is to estimate the impact of household interaction between spouses on employment and participation in the workforce. As explained above, the household types, i.e. Modern, Classical and Cooperative, are viewed as exogenously determined by social or cultural norms. The question is whether we can link social norms to the employment of husbands and wives. We first compare the Classical household to the Modern and then the Modern to the Cooperative. Note that we assume that preferences and market opportunities are identical in all three types of households and therefore any differences in employment patterns can only be due to the nature of the game.

Husbands in the Classical and Modern households have the same optimization problem and the same information set when decisions are made and therefore we would expect them to make similar choices. In contrast, the wives have a different information set in each household. Thus, in the Classical household the wife knows her husband's employment outcome and wage and chooses to enter the labor force only if his actual wage is "too low". In contrast, the Modern wife does not know her husband's

outcome and wage and her decision is not made in response to her husband's employment situation. Therefore, we would expect there to be a strong negative correlation between the employment situations of husbands and wives in Classical households, while in the Modern household the correlation will be weaker.

An important and intuitive implication of the model is that wives in Modern households work more than those in Classical households, even though there is no difference in the women's employment parameters and participation choices. However, we were not able to prove this result as a general analytical outcome and therefore simulations of a two-period model were used in order to nonetheless draw some conclusions. Based on the simulations, we found that for a female in a Modern household to work as much or more than a female in a Classical household, **one** of the following two sufficient conditions must be fulfilled:<sup>18</sup>

1. Women earn less than men, with all other parameters being equal.<sup>19</sup>
2. Women are more risk averse than men (i.e. have lower  $\gamma$ ), with all other parameters being equal.<sup>20</sup>

Thus, the main result depends on the difference in opportunities (wages) and preferences between men and women. The first condition ensures that the leader (i.e. the husband) in the Classical household will usually search. Note that if the wife's income is higher than her husband's, the husband might choose not to search, knowing that his wife will react to his decision by searching. If condition 1 is not fulfilled, we might obtain the undesired result that Classical husbands work less than Modern husbands

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<sup>18</sup> The sufficient conditions hold for certain reasonable values of the model's parameters. A full description of the results can be found at [www.tau.ac.il/~eckstein/HLS/HLS\\_index.html](http://www.tau.ac.il/~eckstein/HLS/HLS_index.html).

<sup>19</sup> For a low job-offer probability (0.7 or less), a wage gap of only 3 percent induces the Classical female to search only if her husband is unemployed while the Modern female always searches. For a higher probability, a larger wage gap is needed for this to occur.

<sup>20</sup> The combination of a lower wage, a lower job-offer probability and higher risk aversion produces similar results.



and Classical wives work more than Modern wives.<sup>21</sup> The second condition implies that a more risk-averse wife in a simultaneous decision game (i.e. in a Modern household) will work more than if she was reacting to her husband's actual observed outcomes (as in a Classical household) since she is uncertain of the result of her husband's search. The Classical wife will work more (or at least not less) than the female in the Modern household only if the male is unemployed or ends up earning less than expected.

The key difference between the Cooperative household and the other types is that the spouses jointly maximize their utility. Thus, the wife may prefer that her husband work less and that she work more. This trade-off does not exist in either the Modern or Classical households. For the same parameter values and the same wages, individuals in a Cooperative household have higher utility from employment through their spouse's utility. In other words, in addition to the direct utility from his/her own wage the individual will have additional indirect utility through his spouse's utility from his/her own wage. For example, suppose that a wife would choose not to work based on her own individual utility. In a Cooperative household, however, she would also evaluate the marginal gain of her husband if she decides to go to work. Since his marginal gain is positive, she may change her employment decision. The case is symmetric for the husband.

### **3. Data and Estimation**

The data is taken from the PSID (Panel Study of Income Dynamics) survey for the period 1983-93. We use quarterly data which is available only from 1983 onward and restrict the model to the first ten years

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<sup>21</sup> Since in the data and in the estimation, the wages and the job offer rates are lower for women, in most Classical households the husband is choosing to search.

of marriage.<sup>22</sup> In order to create similar initial conditions for all individuals, we restrict the data to start from the date of the wedding (in accordance with the model) and consider all couples in the PSID sample who got married during the period 1983-4. The data thus provides information on 863 couples and tracks them until 1993 or until they separate. During the sample period, 36.3 percent of the couples divorced or separated and 14.5 percent were removed from the sample for some other unknown reason, such that after 10 years 49.2 percent of the couples remained in the sample.

The data includes demographic and employment information on individuals and households, such as wages, working hours, unemployment (job search) and non-participation, as presented in Table 1.<sup>23</sup>

**Table 1: Descriptive Statistics**

	PSID DATA		CPS DATA (for comparison)
	End of first year (1984)	End of last year (1993)	1984
<b>Husbands</b>			
Age	30	39.1	30.1
Years of Schooling	12.6	12.8	12.7
Participation Rate	92.6%	93.7%	94.6%
Employment Rate	84.3%	89.9%	84.9%
Hours of work per week	43.2	43.5	43.5
Monthly Salary Income*	1566	4494	1565
<b>Wives</b>			
Age	27.8	36.7	27.8
Years of Schooling	12.7	12.9	12.4
Participation Rate	72.1%	79.1%	60.5%
Employment Rate	67.8%	77.4%	54.9%
Hours of work per week	36.3	34.6	34.3
Monthly Salary Income*	1051	2569	881
# of children	0.8	1.7	1.2
Observations	863	425**	6429

\* US dollars, 1984 prices

\*\* 36.3% divorced, 14.5% dropped out of sample

<sup>22</sup> We solve the recursive optimization backwards from the 11<sup>th</sup> year of marriage and it is assumed to be a parameterized function of the state space in the 40<sup>th</sup> quarter with the terminal value function given by equation (2.10).

<sup>23</sup> For more details on the data, see [www.tau.ac.il/~eckstein/HLS/HLS\\_index.html](http://www.tau.ac.il/~eckstein/HLS/HLS_index.html).

Thus, the employment rate (participation rate) of the women in the sample increased from 67.8 percent (72.1 percent) in 1984 to 77.4 percent (79.1 percent) after 10 years of marriage, while their unemployment rate fell from 5.1 percent to 2.6 percent. The employment rate (participation rate) of the men increased from 84.3 percent (92.6 percent) in 1984, to 89.9 percent (93.7 percent) in 1993 and the unemployment rate decreased from 10 percent to 3.5 percent. During the ten-year period, the average years of schooling and average hours of work remained unchanged. However, real monthly income increased by a factor of 1.87 for men and 1.44 for women. In order to determine whether the PSID sample is representative, we compared it to an equivalent CPS sample, which is presented in the last column of Table 1. The CPS data is restricted to married males and females who were interviewed in 1984 and had the same age distribution as the PSID sample. The main difference between the samples is that the CPS consists of *all* individuals who were married in 1984 while the PSID sample consists only of individuals who were newly married in that year. While the husbands' characteristics and the wives' years of schooling are almost identical in both samples, the couples in the CPS sample have more children and wives' participation and employment rates are lower. This is not surprising, given the shorter time that couples in the PSID sample have been married.

**Table 2: Wives' employment states conditional on their husbands' employment states**

<b>Husband's Labor State</b>	<b>Wife's Labor State</b>		
	Employed	Unemployed	Out of Labor Force
Employed	75.4%	3.5%	21.0%
Unemployed	64.5%	6.5%	29.0%
Out of Labor Force	65.0%	3.4%	31.6%

Table 2 presents the employment states of wives conditional on their husbands' labor market state. It is interesting to note that the employment rate (out-of-labor-force rate) among women is 75.4 percent (21 percent) if their husband is working but only about 65 percent if he is unemployed or out of the labor force. In other words, a woman is more likely to be employed if her husband is employed than if he is unemployed or out of the labor force. To account for this in a model where both spouses endogenously determine their labor supply is an additional challenge to be dealt with.

### ***Estimation***

The model is estimated using standard SMM (Simulated Method of Moments) following Pakes and Pollard (1989) and McFadden (1989). Estimation involves solving the model three times, i.e. once for Modern households, once for Classical households and once for Cooperative households, where the value of the objective function is calculated separately for each member of each type of household. We relate to the household type as unobserved heterogeneity where the probabilities are functions of the couple's characteristics on the day of the wedding: the partners' ages, education and assortative mating, as measured by education and age gaps. The probability of being a Classical household is given by:

$$(3.1) \quad \pi_{C_i} = \exp(TYPE\_C_i) / (1 + \exp(TYPE\_C_i) + \exp(TYPE\_M_i))$$

where

$$\begin{aligned} TYPE\_C &= \gamma_1^C \cdot AGE_1^W + \gamma_2^C \cdot AGE_1^H + \gamma_3^C \cdot S_1^W + \gamma_4^C \cdot S_1^H + \gamma_5^C \cdot (AGE_1^W - AGE_1^H)^2 + \gamma_6^C \cdot (S_1^W - S_1^H) \\ TYPE\_M &= \gamma_1^M \cdot AGE_1^W + \gamma_2^M \cdot AGE_1^H + \gamma_3^M \cdot S_1^W + \gamma_4^M \cdot S_1^H + \gamma_5^M \cdot (AGE_1^W - AGE_1^H)^2 + \gamma_6^M \cdot (S_1^W - S_1^H) \end{aligned}$$

The probability of being a Modern or Cooperative household has a symmetric form. Let  $T_i$  be the length of time we observe household  $i$  and let  $\theta$  be the vector of parameters, including the parameters in  $\Sigma$ . We denote the data on spouses' actual choices in household  $i$  as  $(d_{ij}^o; t=1, \dots, T_i; j=W, H)$  and the predicted choices for a household of type  $h = M, C, COP$  as  $(d_{ij}^p(h, \theta); t=1, \dots, T_i; j=W, H)$ . We define:

$$(3.2) \quad \begin{aligned} D_{ij}^h(\theta) &= 0 \quad \text{if } d_{ij}^o = d_{ij}^p(h, \theta) \\ D_{ij}^h(\theta) &= 1 \quad \text{otherwise} \end{aligned}$$

$D_{ij}^h(\theta)$  equals zero if the model correctly predicts the choice of individual  $j$  in household  $i$  in period  $t$  under the specification of family type  $h$  and one otherwise. Hence,  $D_{ij}^h(\theta)$  is a matrix of moments that includes the predicted and observed transition probabilities. We weight  $D_{ij}^h(\theta)$  according to the

household type proportion estimated using (3.1). The weighted average of the three household types according to the assumed proportions, i.e.  $\pi_{Ci}$ ,  $\pi_{Mi}$  and  $(1 - \pi_{Ci} - \pi_{Mi})$ , is given by:

$$(3.3) \quad g_{1i}(\theta) = \pi_{Ci} \sum_{t=1}^{T_i} \sum_{j=H,W} D_{ij}^C(\theta) + \pi_{Mi} \sum_{t=1}^{T_i} \sum_{j=H,W} D_{ij}^M(\theta) + (1 - \pi_{Ci} - \pi_{Mi}) \sum_{t=1}^{T_i} \sum_{j=H,W} D_{ij}^{COP}(\theta)$$

The sum of these elements for all households is the first moment to be minimized and is given by:

$$(3.4) \quad g_1(\theta) = \sum_{i=1}^{863} g_{1i}(\theta)$$

We denote the actual wage of the individual as  $(w_{ij}^o; t = 1, \dots, T_i; j = W, H)$  and the predicted equivalent for a household of type  $h$  as  $(w_{ij}^p(h, \theta); t = 1, \dots, T_i; j = W, H)$ . The second set of moments is based on the difference between observed and predicted wages. Specifically, we calculate the squared difference between the average over households of the observed and predicted weighted wage per household in each quarter  $t$  for H and W separately. The average weighted wage of the three household types is

$$(3.5) \quad \bar{w}_j^p(\theta, \pi_C, \pi_M) = \pi_C \bar{w}_j^p(C, \theta) + \pi_M \bar{w}_j^p(M, \theta) + (1 - \pi_C - \pi_M) \bar{w}_j^p(COP, \theta)$$

where  $\pi_C, \pi_M$  and  $(1 - \pi_C - \pi_M)$  are the average proportions of the household types over the observed sample and  $\bar{w}_j^p(h, \theta)$  is the average over  $i$  of the simulated wage. There are forty (40) quarters of data and we calculate moments for H and W, such that there are 80 wage moments. Let  $g_2(\theta, \pi_C, \pi_M)$  be the vector of these 80 moments as follows:

$$(3.6) \quad g_2(\theta, \pi_C, \pi_M) = [(\bar{w}_{1H}^o - \bar{w}_{1H}^p(\theta, \pi_C, \pi_M))^2, \dots, (\bar{w}_{40H}^o - \bar{w}_{40H}^p(\theta, \pi_C, \pi_M))^2, \dots, (\bar{w}_{1W}^o - \bar{w}_{1W}^p(\theta, \pi_C, \pi_M))^2, \dots, (\bar{w}_{40W}^o - \bar{w}_{40W}^p(\theta, \pi_C, \pi_M))^2]$$

We define the vector of moments as  $g(\theta, \pi_C) = [g_1(\theta, \pi_C), g_2(\theta, \pi_C)]$ . The SMM is defined by the minimum of the objective function:

$$(3.7) \quad J(\theta, \pi_C, \pi_M) = g(\theta, \pi_C, \pi_M)' W g(\theta, \pi_C, \pi_M)$$

with respect to  $\theta$  and  $\pi_C, \pi_M$ , where the weighting matrix  $W$  is a diagonal matrix. The weight assigned to each moment is the inverse of the estimated standard deviation of the specific moment in the data. We find the estimated standard errors using the inverse of the Jacobian matrix.<sup>24</sup>

## 4. Results

The estimation results enable us to determine whether there is indeed more than one type of household. The estimated proportion of Classical households is 0.573 and that of Modern households is 0.254 (see Table 5 for main parameters). Furthermore, estimating the model by assuming that all households solve the Classical game increases the  $J$  value from 48.63 to 211.7. Assuming that all households solve the Modern game increases the  $J$  value to 421.5 and assuming that all households solve the Cooperative game increases the  $J$  value to 445.8. Hence, using the standard test statistic (Newey and West, 1987) we reject the hypothesis that all households follow the same type of game in determining the couple's labor supply.<sup>25</sup> On the other hand, we could not reject the hypothesis that there are only two types of households (see table 3) and that one of them is Classical.

The identification of types is based primarily on the identification of unobserved heterogeneity, which is similar to identifying fixed effects in panel data. In the SMM estimation, it is based on the moments of the transition matrix. Thus, the result that the Classical household type cannot be excluded from the model is based on the transition matrix data. The data show that women tend to enter the labor force after their husbands have left the labor force or become unemployed. This transition is consistent with the specification of the Classical household in which women react to the husband's outcomes. As a

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<sup>24</sup> The size of the state space is about three hundred millions (298,252,800) which required about 1.2 GB of memory. Each element in the state space is calculated thirty times. The number of parameters is 78. This puts an enormous burden on the computational complexity of the model. To estimate the model we used advanced programming routines and parallelization.

<sup>25</sup> It should be noted that one could allow for a more flexible form of Classical game (e.g., unobserved heterogeneity in utility and other parameters) in which case the hypothesis of one type of household may not be rejected.

result, when the Classical family was excluded from the model, we could not match those moments well and the restricted specification was rejected.

**Table 3: Types Identification**

<b>Restricted Objective Function</b>	
<b>Only one Type</b>	
Classical	211.7
Modern	421.5
Cooperative	445.8
<b>Two Types</b>	
Classical and Modern	55.2
Classical and Cooperative	61.6
Modern and Cooperative	98.3
<b>Unrestricted Objective Function</b>	
<b>Three types</b>	48.6

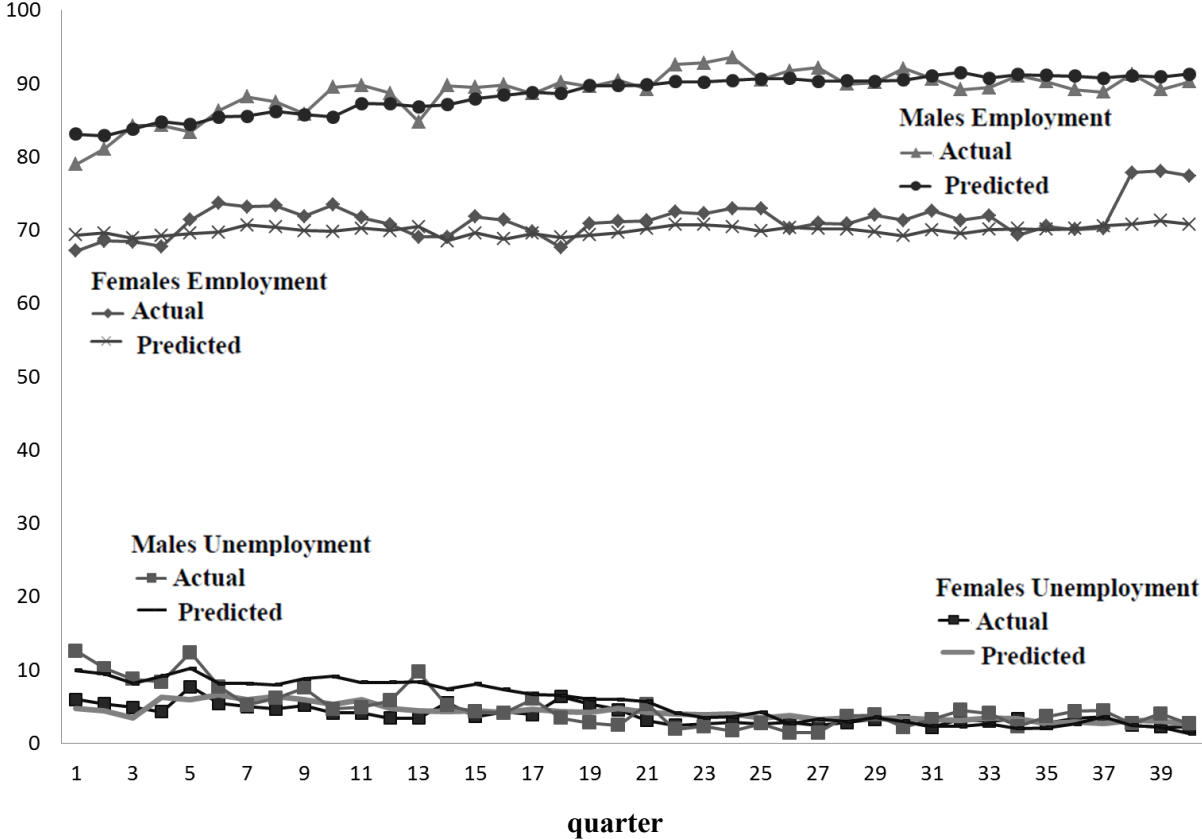
In what follows, we first look at how well the estimated model fits the observed average employment states, the transitions between states and average wages by gender, conditional on the estimated parameters. Given that the model provides a good fit to the data, we then interpret the estimated parameters. Finally, we use the estimated model to simulate counterfactual predictions (both within-sample and out-of-sample) for the labor supply of the three types of households.

### ***Goodness of Fit***

The estimated parameters and assumed random errors were used to calculate the predicted proportions of the three labor market states in the sample. The calculations were carried out for all observed households which were each classified as Modern, Classical or Cooperative and averaged using the estimated proportions of household type. Figure 1 presents the actual and the predicted proportions of men and women in the states of employment (E) and unemployment (UE). The estimated model provides a good fit to the aggregate proportions and a simple goodness-of-fit test for each choice over the entire sample gives a value which is under the critical 5 percent level for all cases, except UE

for men and women.<sup>26</sup> We also tested the goodness-of-fit of actual to predicted choices for each of the 40 quarters of data and in 34 (36) of the 40 quarters, the model passes the simple  $\chi^2$  goodness-of-fit test for women (men).<sup>27</sup>

**Figure 1: Actual and Predicted Choice Distribution**



The model accurately predicts the trends and levels of actual wages for both men and women, except for the large outlier in actual real wages in 1993, which is the last year of the sample (see Figure 2).<sup>28</sup>

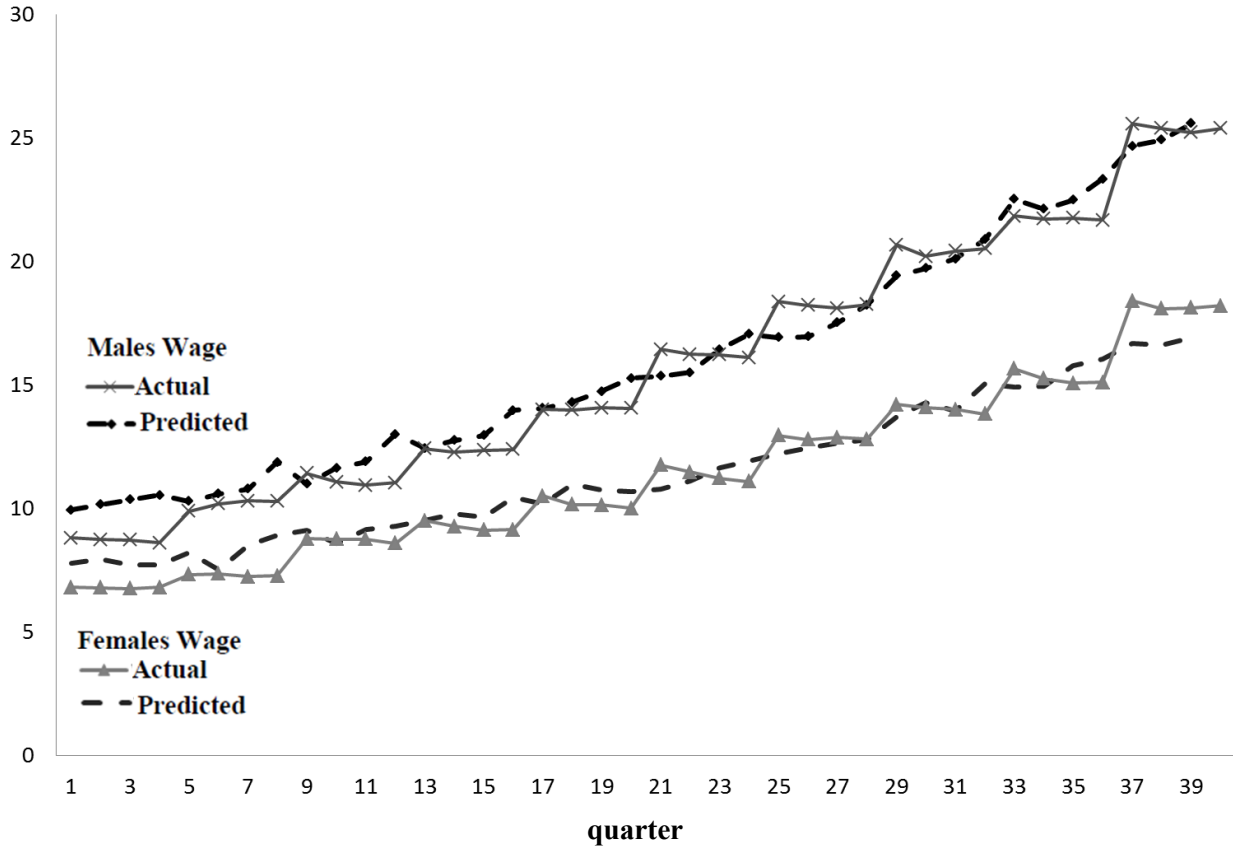
<sup>26</sup> The chi-square test statistics for employment, unemployment and out of the labor force are 9.56, 288.3 and 52.09, respectively, for males and 19.99, 77.36 and 52.63, respectively, for females. The relevant critical value is  $\chi^2(39) = 54.57$ .

<sup>27</sup> For the full results, see [www.tau.ac.il/~eckstein/HLS/HLS\\_index.html](http://www.tau.ac.il/~eckstein/HLS/HLS_index.html).

<sup>28</sup> There are only 425 observations for the last year.



**Figure 2: Actual and Predicted Mean Wages**



Using a simple t-test for the equality of mean predicted wages for men and women we cannot reject the hypothesis that estimated and actual means are equal for the entire sample. Using the same test separately for each period, we reject the hypothesis for some periods.<sup>29</sup> In Table 4, we report the predicted distribution of the wives’ labor market states conditional on their husbands’, both in the aggregate and by type of household. The predicted aggregate distribution displays the identical pattern and its values are close to those of the actual distribution (Table 2). In other words, the estimated model captures the positive correlation between husband and wife in each of the three labor market states: employment, unemployment and out of labor force. This observation reflects the similarity of labor market opportunities for husbands and wives (e.g., as a result of assortative mating) rather than

<sup>29</sup> For all periods combined, the *t*-test statistic is 0.71 for males and 0.74 for females. In separate tests for each period, the hypothesis is rejected in the case of women for periods 1-4, 6, 8 and 19 and in the case of men for periods 2, 3, 4 and 29.

potential income insurance motives. The correlation by type of household differs from that of the aggregate and this will be discussed below when we look at employment by type of household.<sup>30</sup>

**Table 4: Wives' estimated employment states conditional on their husbands' employment states by family type**

<b>Husband's Labor State</b>	<b>Wife's Labor State</b>		
	Employed	Unemployed	Out of Labor Force
Employed	<b>73.4%</b>	<b>4.1%</b>	<b>22.5%</b>
Classical families	63.0%	2.3%	34.7%
Modern families	80.1%	5.2%	14.7%
Cooperative families	85.3%	6.2%	8.6%
Unemployed	<b>66.4%</b>	<b>6.2%</b>	<b>27.4%</b>
Classical families	66.2%	7.1%	26.7%
Modern families	65.6%	4.0%	30.5%
Cooperative families	71.8%	4.8%	23.4%
Out of Labor Force	<b>66.2%</b>	<b>4.4%</b>	<b>29.4%</b>
Classical families	66.8%	4.9%	28.3%
Modern families	63.0%	3.7%	33.2%
Cooperative families	73.7%	3.6%	22.7%

**Parameters (Table 5)**

Women are more risk averse than men as can be seen from the risk aversion parameter ( $\gamma_w = 0.89$  for women and  $\gamma_H = 0.97$  for men).<sup>31</sup> Furthermore, women attribute a higher value to leisure (home production) than men (1.0 vs. 1.2). Labor search costs are positive and the joint family parameters of utility from children ( $\gamma_1$  and  $\gamma_2$ ) have the expected signs (i.e. positive) and magnitudes. As one might expect, the quality and quantity of children have a larger effect on women's utility than on men's (1.12 vs. 0.98).

<sup>30</sup> It is worth mentioning that the good fit of the estimated model to the data is not a complete surprise since these moments were used for the SMM estimation criterion.

<sup>31</sup>  $u_j(x_t) = (x_t)^{\gamma_j} / \gamma_j$

**Table 5: Estimated Parameters**

Utility*			Wage**			Job Offer Probability***		
	Male	Female		Male	Female		Male	Female
$\gamma_j$ - risk aversion	0.97 (0.066)	0.89 (0.083)	$\beta_1$ - constant	0.577 (0.208)	0.426 (0.085)	$\rho_{01}$ - employed at t-1	2.787 (0.246)	2.75 (0.335)
$\alpha_j$ - value of leisure	1.002 (0.138)	1.232 (0.117)	$\beta_2$ - experience	0.033 (0.008)	0.03 (0.014)	$\rho_{02}$ - unemployed at t-1	1.598 (0.246)	1.375 (0.337)
utility from childrens	0.979 (0.44)	1.12 (0.092)	$\beta_3$ - experience <sup>2</sup>	-0.000001 (0.0000005)	-0.000002 (0.0000003)	$\rho_{03}$ - OLF at t-1	0.545 (0.064)	0.35 (0.073)
search cost	0.658 (0.179)	0.602 (0.045)	$\beta_4$ - schooling	0.08 (0.008)	0.083 (0.012)	$\rho_1$ - schooling	0.085 (0.044)	0.096 (0.018)
unemployment benefit	2.632 (0.383)	2.515 (0.27)				$\rho_2$ - experience	0.07 (0.005)	0.021 (0.002)
Type Proportion****			utility from quality and quantity of children*****					
	Classical	Modern	CES parameter	0.19 (0.042)				
wife's age	0.09 (0.005)	-0.126 (0.013)	wife's leisure	0.127 (0.028)				
husband's age	0.07 (0.005)	-0.034 (0.002)	husband's leisure	0.094 (0.048)				
wife's schooling	-0.285 (0.031)	0.22 (0.041)	wife's leisure when infant	0.05 (0.013)				
husband's schooling	-0.254 (0.019)	0.122 (0.007)	husband's leisure when infant	0.013 (0.015)				
age gap	0.326 (0.032)	-0.127 (0.008)	spending per child	0.487 (0.152)				
schooling gap	0.324 (0.017)	0.151 (0.019)						

Standard errors appear in parentheses \* see equations 2.1, 2.5 and 2.6 (note that  $\gamma_0$  is unidentified). \*\* see equation 2.2. \*\*\* see equation 2.7. \*\*\*\* see equation 3.1 \*\*\*\*\* see equation 2.4

The wages of both men and women increased substantially during the sample period (Figure 2). As a result, the estimated experience parameters in the wage equation are large and slightly higher for the husband than for his wife. Interestingly, the estimated rate of return on a year of schooling is slightly lower for the husband than for his wife (0.08 vs. 0.083). In the sample, men have somewhat less years of schooling than women (12.7 vs. 12.8). The constant is higher for men than for women and therefore the expected wage offer for a newlywed man is higher than that for his newlywed wife unless she has significantly more years of schooling. This situation is not a common one given the assortative mating observed in the data (i.e. a correlation of 0.52 in years of schooling between husband and wife).

The job-offer probability parameters are higher for men than for women, apart from the education parameter (see Table 5).<sup>32</sup> In particular, males have higher job offer rates when they are unemployed and out of the labor force. In light of their higher job offer rates and higher wage offers conditional on the labor market state, husbands' job market opportunities are superior to those of their wives.

<sup>32</sup> The other parameters are presented in Table 5.

The household type probability parameters suggest that the years of schooling of both partners increase the probability of the household being Modern and decrease the probability of it being Classical. Classical households are older and M households are younger than Cooperative households. The level of assortative mating (with respect to both age and education) is higher for Modern households and lower for Classical households than for Cooperative households. Thus, Modern households are younger and have more education and the characteristics of the couple are more similar. Classical households are older and less educated in general; however, if one of the spouses is much younger or less educated than the other the household is more likely to be a Classical household. The Cooperative households are somehow in between the Classical and Modern households. The average proportion of Classical households is 57.3 percent, that of Modern households is 25.4 percent and that of Cooperative households is 17.2 percent.

The parameters of the exogenous processes of having children and divorce have the predicted signs (Table 6).

**Table 6: Additional Estimated Parameters**

<b>Probability of Another Child*</b>		<b>Probability of Divorce**</b>		<b>Error Covariance Matrix</b>	
W worked at t-1	-0.019 (0.002)	marriage duration	0.004 (0)	L(1,1)	0.497 (0.033)
H worked at t-1	0.08 (0.008)	marriage duration square	-0.001 (0)	L(2,1)	1.279 (0.074)
Wage	0.054 (0.007)	W schooling	-0.043 (0.022)	L(2,2)	1.152 (0.078)
Wage squared	-0.005 (0)	H schooling	-0.086 (0.044)	L(3,1)	0.435 (0.055)
H age	0.058 (0.03)	# of children	-1.776 (0.111)	L(3,2)	1.31 (0.194)
W schooling	-0.006 (0.004)	<b>Terminal Value***</b>		L(3,3)	1.223 (0.191)
H schooling	- (0.001)		Males	Females	
# of children	-0.858 (0.176)	W schooling	1.328 (1.232)	9.064 (4.693)	
existence of an infant	-2.626 (0.254)	W exp	1.027 (1.293)	3.004 (0.162)	
		H schooling	8.325 (4.502)	4.26 (2.806)	
		H exp	3.455 (0.172)	2.659 (1.427)	
		worked at t-1	10.394 (4.883)	9.655 (3.855)	

Standard errors appear in parentheses. \* see equation 2.8 \*\* see equation 2.9 \*\*\* see equation 2.10

The probability of having another child decreases with number of children, the existence of an infant (ages 0-3), the parents' level of schooling and if the wife was employed in the previous quarter and increases with the ages of the parents. The probability of divorce increases (at a decreasing rate) with years of marriage and decreases with the couple's level of education and number of children. Terminal values, estimated parameters and the estimated variance matrix of the three errors are presented in Table 6.

### ***Employment by Type of Household***

The estimated parameters are consistent with the assumption that the husband's labor market opportunities and incentives are superior to those of his wife and therefore his search intensity is greater. As a result, the employment rate of husbands is much higher for all household types (Figure 1). Simple chi-square tests indicate that the employment state distributions of Classical and Modern women are significantly different in all 40 quarters, based on Figure 3A results. We cannot reject the hypothesis that employment and unemployment rates are the same for Modern and Cooperative households.<sup>33</sup> This result is consistent with the above result that either Modern or Cooperative types can be omitted (Table 3).

The average employment rates of wives in the Modern and Cooperative households is 76 and 80 percent, respectively, which are much higher than the rate for Classical wives (i.e. 64 percent). Furthermore, the unemployment rates of wives in Modern households (5.2) and Cooperative households (5.6) are also higher than for wives in Classical households (3.3) which is an indication that their search intensity is higher. These results derive from the assumptions that a wife in a Modern or Cooperative household searches simultaneously with her husband while the wife in a Classical household reacts to

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<sup>33</sup> The chi-square test statistics for employment, unemployment and out of the labor force between women in Classical and Modern households are 580.3, 593.0 and 1528.8, respectively. The chi-square test statistics for employment, unemployment and out of the labor force between women in Modern and Cooperative households are 54.7, 46.3 and 294.5, respectively. The critical value is  $\chi^2(39) = 54.57$ . For the full results, see [www.tau.ac.il/~eckstein/HLS/HLS\\_index.html](http://www.tau.ac.il/~eckstein/HLS/HLS_index.html).

the outcome of her husband's search. According to the results, the wife has significant risk aversion and the husband's unemployment and out-of-labor-force rates are relatively low for all households. Given the unknown outcome of her husband's search, the Modern and Cooperative wives search more intensively than the wives in Classical households.

The correlation between the employment level of the spouses is positive (Table 4), which means that when the husband is employed the wife has a higher probability of also being employed. This is true in the actual data and in the results for the Modern and Cooperative households. Note also that when the husband in those households is unemployed or out of labor force his wife's employment rate drops by an average of 15 percent relative to the state when the husband is employed. In contrast, we obtain the opposite result for Classical households. The employment rate of Classical wives is 63 percent when her husband is employed, which increases to 66 percent when her husband is unemployed or out of labor force. In the Classical household, the element of income insurance is much stronger as a result of the sequential decision making. As mentioned above, assortative mating in the form of similarity in education and age implies a positive correlation between the labor market outcomes of a husband and wife. According to the estimation results, assortative mating is least prevalent in Classical households, which explains the negative correlation for that type (see equation (3.1) and Table 5).

Figure 3A: Predicted Choice Distribution by Type - Female

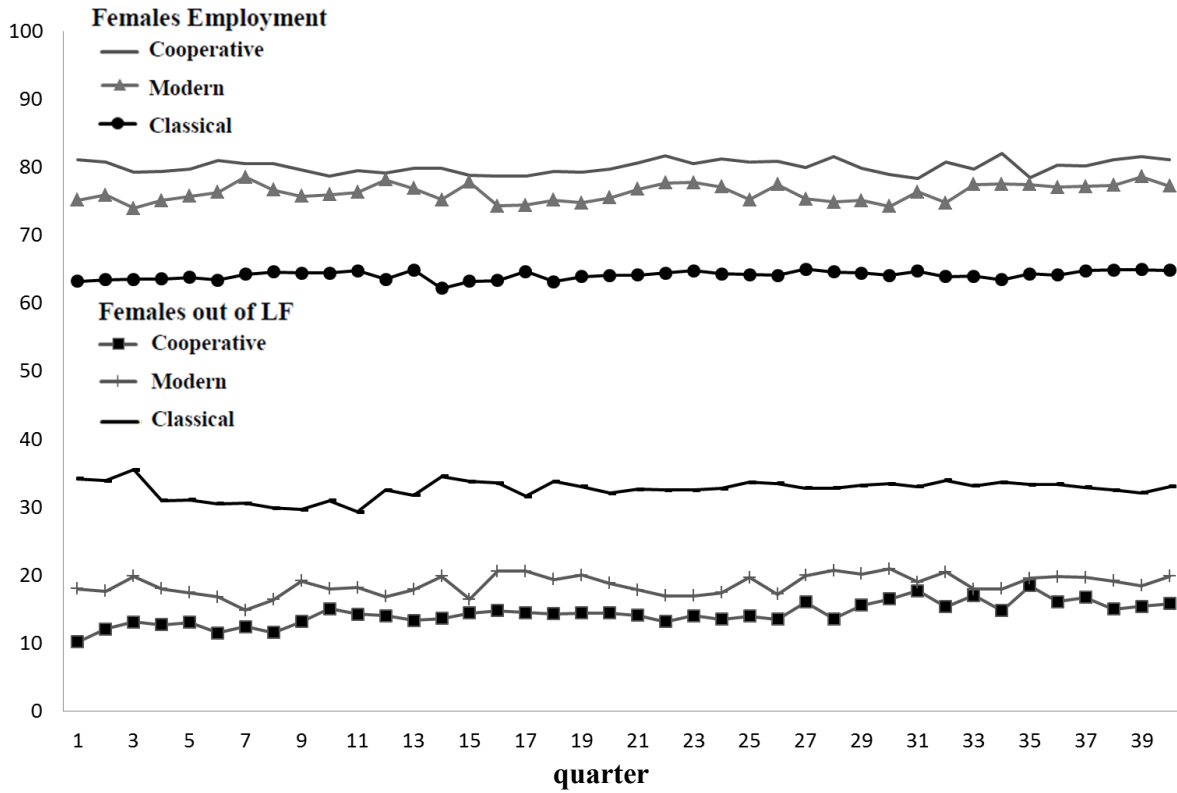
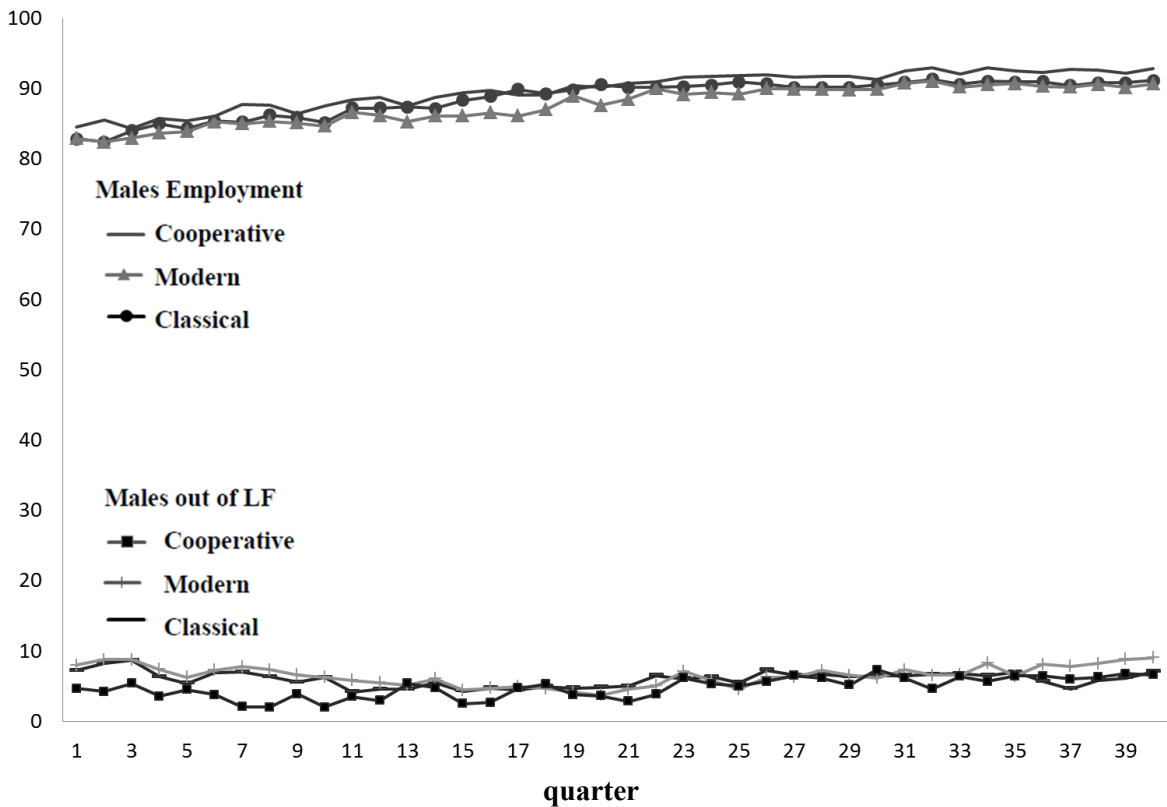


Figure 3B: Predicted Choice Distribution by Type – Male



As mentioned, husbands' employment rates are similar in the three types of households (88.6 percent for Classical, 87.7 percent for Modern and 89.8 for Cooperative) and consequently their unemployment and out-of-labor-force rates are similar. Chi-square tests showed that there are no significant differences in predicted employment rates between husbands in the three types of households in any of the 40 quarters.<sup>34</sup> This result is due to two aspects of the model and the estimated parameters: First, the husband has a very low estimated level of relative risk aversion ( $\gamma_H = 0.97$ ), such that he is essentially indifferent to his wife's impact on household consumption. Hence, a potential change in a wife's labor supply does not significantly affect the husband's decisions in any of the household types and therefore the game structure is irrelevant to the husband's labor supply. Second, the male's decisions in all the games are based on the same information regarding female employment opportunities. Thus, even with a higher degree of risk aversion one would expect that men's employment outcomes will differ less by type of household than women's. In the Cooperative household, the incentive to work is higher since wages enter the objective function directly (through consumption) and indirectly (through the partner's utility), which explains the higher employment rate of both partners in this type of household.<sup>35</sup>

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<sup>34</sup> The chi-square test statistics for employment, unemployment and out of labor force between men in Classical and Modern households are 4.5, 125.9 and 51.5 respectively and between men in Modern and Cooperative households they are 14.0, 114.1 and 235.3 respectively. The critical value is  $\chi^2(39) = 54.57$ .

<sup>35</sup> For the Cooperative household, we used a Bargaining Power (BP) parameter of 0.5. We simulated the model using BP's of zero and one, which is equivalent to a dictator game, once with the wife as the dictator and once with the husband. Interestingly, though not surprisingly, when the woman is the dictator the husband always searches,  $d_{IH}^* = 1$  for  $t = 1, \dots, 40$ , and the wife's employment rate drops to below the benchmark (0.5 BP). In general, the higher the BP of the wife the lower is her employment rate and similarly for the husband. The highest employment rate for women in the model was in the case where the husband is the dictator (BP=1).



One method of analyzing the empirical content of the estimated unobserved household types is to make use of the correlation between the estimated type probability of each household conditional on the observed employment outcomes (i.e. the posterior probability; see, for example, Eckstein and Wolpin, 1999) and household demographic indicators not included in equation (3.1). Table 7 shows that a Classical couple tends to have more children and to live in a rural area and that is more likely that the head of the household is Protestant and that the marriage is stable (i.e. the couple is less likely to divorce).

**Table 7: Correlation between Posterior Type Probability and Household Characteristics**

Variable	Estimated Probability of		
	C Household	M Household	COL Household
# of children in household	0.369	-0.251	-0.330
White husband	-0.048	0.080	0.235
Afro-American husband	0.090	-0.077	0.025
Catholic husband	-0.051	0.066	0.070
Protestant husband	0.045	-0.031	-0.002
Divorced during sample period	-0.106	0.129	-0.110
Residence in a city	0.014	0.006	0.212
Residence in a small town	-0.019	-0.010	0.005
Residence in a rural area	0.063	-0.055	0.008

The probability that a Modern couple stays married for 10 years is lower than for a Classical or Cooperative couple and they are likely to have fewer children. The Cooperative couple tends to have less children relative to the Classical household, is more likely to live in a city and is more likely to have a white Catholic head of the household. These results are consistent with our a-priori probabilities on the demographic characteristics of Classical, Modern and Cooperative households and therefore our confidence in the model's interpretation of the data is reinforced.

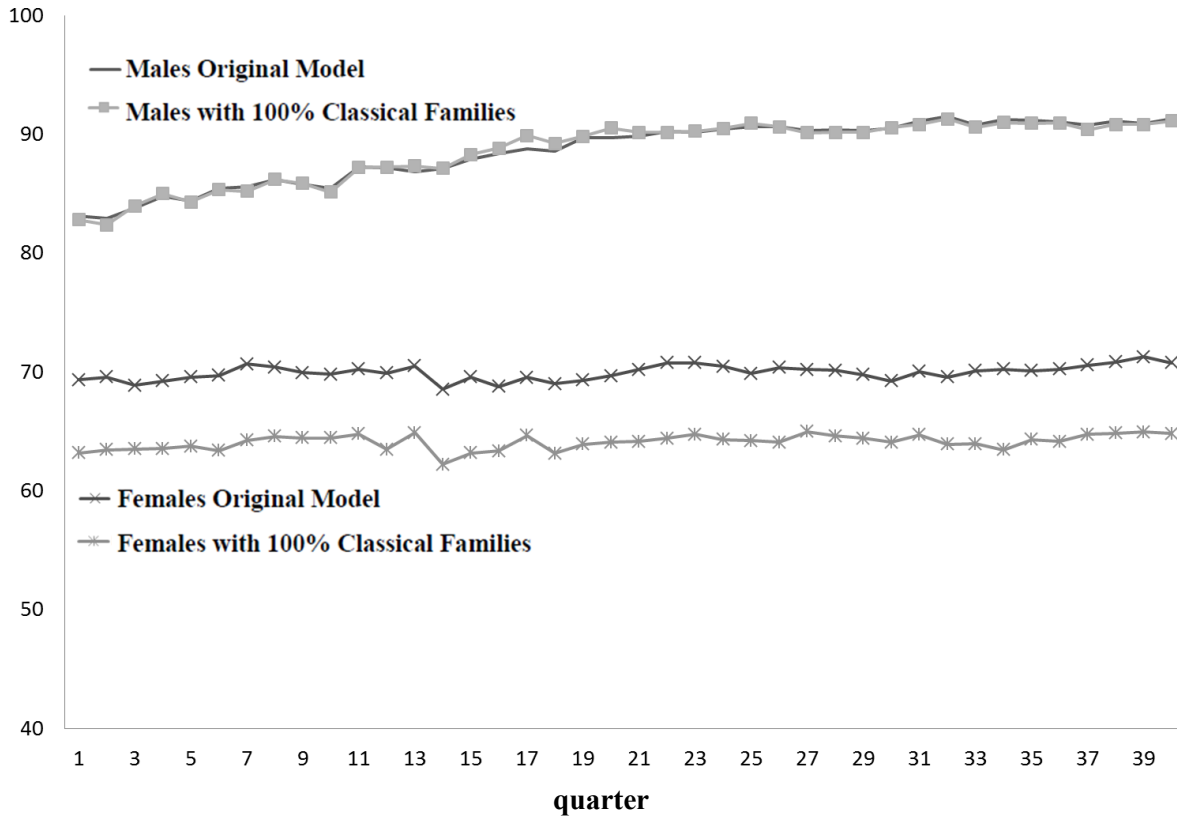
## 5. Counterfactuals

In this section, we use the estimated model to measure the potential increase in female employment rates due to a change in the rules of the game, i.e. in social norms, which determine the household's joint labor supply. This is accomplished through five simulations in which all households are of the same type: in the first three, we assume that all households are of the same type (either Classical, Modern or Cooperative) and in the last two we assume that all households are of the same type (either Modern or Cooperative) and that employment opportunities are identical for men and women in terms of wages and job-offer rates.

### *Simulation 1: All households are Classical (Figure 4)*

We assume that all households are Classical rather than the estimated proportion of 57 percent. As a result, the average predicted female employment rate decreases to 0.64 from the estimated rate of 0.70 while the predicted male employment rate remains almost the same (0.886). The decrease of 6 percentage points in the employment rate of women is due to the fact that women with employed husbands who choose to search under the Modern and Cooperative specification tend to choose not to under the Classical specification.

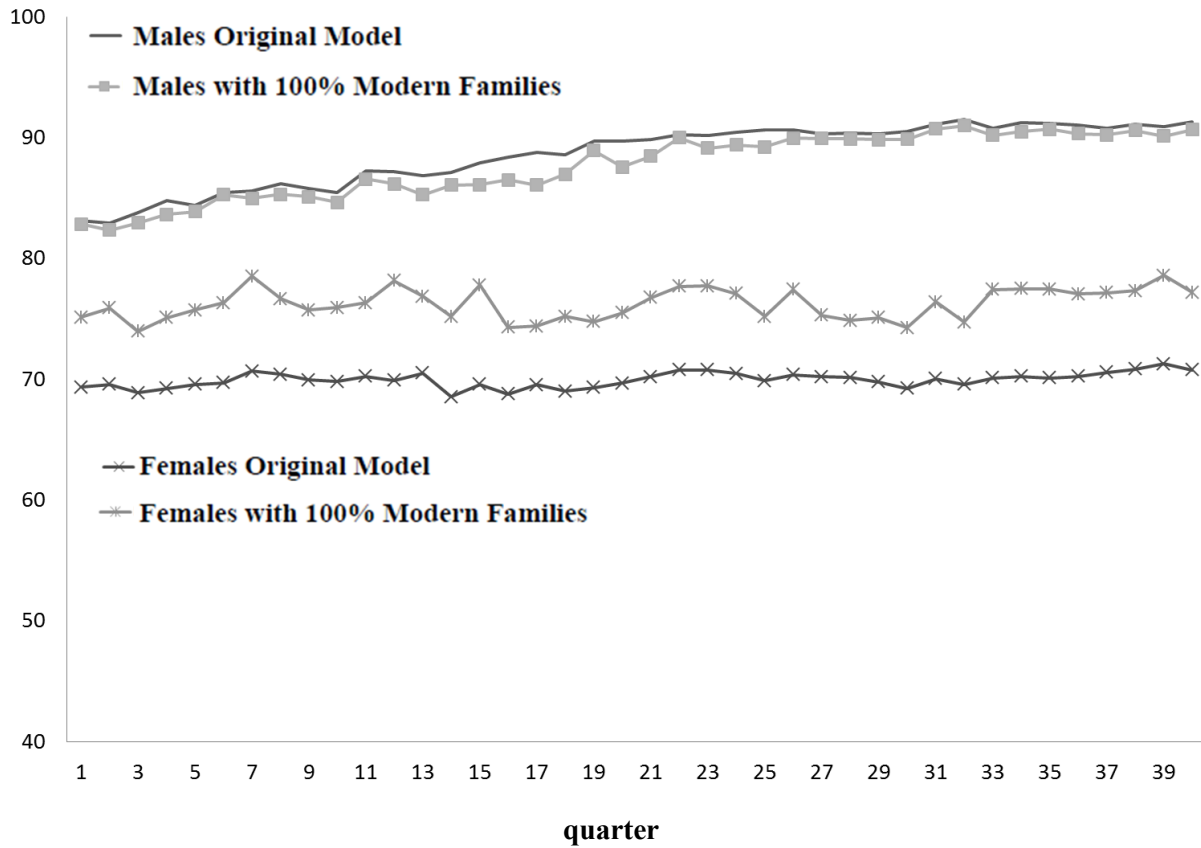
**Figure 4: Simulation 1 - Predicted Employment Rate with 100 percent Classical Families**



***Simulation 2: All households are Modern (Figure 5)***

We assume that all households are Modern rather than the estimated proportion of only 25.4 percent. As a result, the predicted female employment rate increases to 76 from the estimated 70 while the predicted male employment rate decreases slightly (to 88 as compared to the estimated rate of 89). According to the predicted outcome of the simulation, even when the entire population consists of Modern households the male employment rate exceeds that of women by 11.5 percentage points. This is due to the differences in wages, job-offer probabilities and preferences, as explained above.

**Figure 5: Simulation 2 - Predicted Employment Rate with 100% Modern Families**

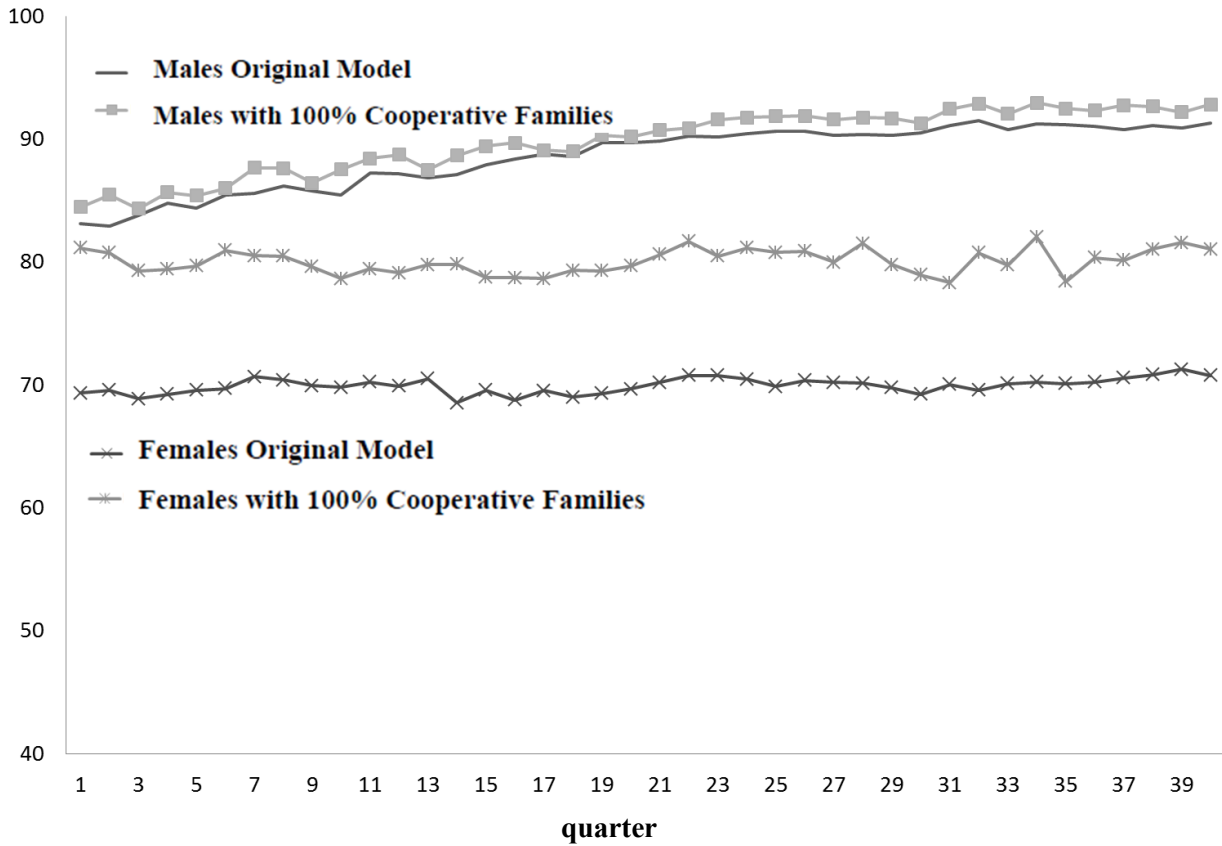


**Simulation 3: All households are Cooperative (Figure 6)**

We assume that all households are Cooperative rather than the estimated proportion of only 17.2 percent. As a result, the predicted female employment rate increases to 0.80 from the estimated 0.70 while the predicted male employment rate increases slightly (to 0.90 as compared to the estimated rate of 0.89). As in the previous simulation, even when the entire population consists of Cooperative households the male employment rate exceeds that of women by 9.8 percentage points.

The results of simulations 2 and 3 imply that changes in social norms over time, as represented by a change in the proportions of Modern and Classical households for different cohorts, may have had a large impact on the employment rate of married women, while hardly affecting that of married men. This result is consistent with the data (Eckstein and Lifshitz, 2011).

**Figure 6: Simulation 3 - Predicted Employment Rate with 100% Cooperative Families**



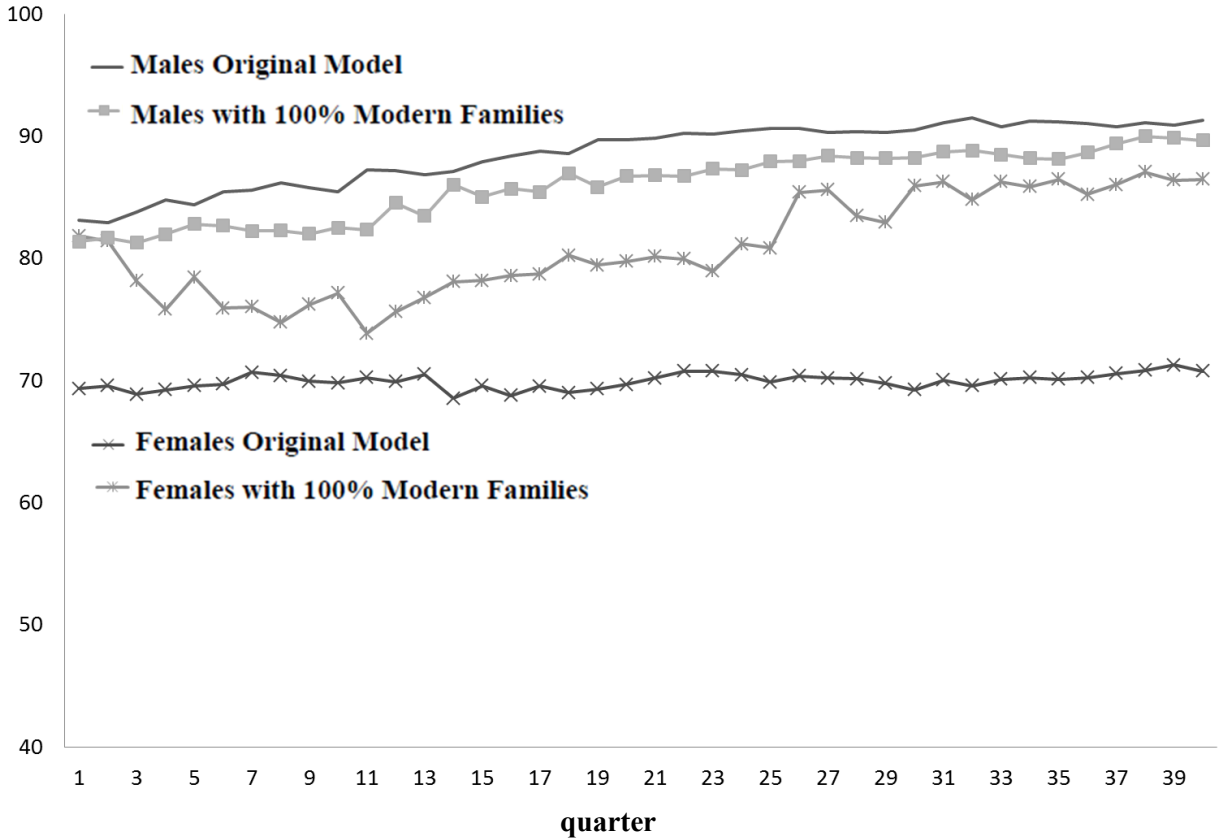
*Simulation 4: All households are Modern and employment opportunities for both genders are identical (Figure 7)*

In addition to the assumptions of Simulation 2, we now calibrate the female wage function and job-offer probability parameters to the values estimated for men. As a result, the employment rate of women increases to 0.81 and that of men decreases to 0.86. Thus, male and female employment rates differ by only 5 percentage points in this case, which is due solely to differences in the utility function parameters. For example, the value of leisure is higher for women (\$1.2 dollars per hour for women as compared to only \$1 per hour for men). In addition, women have a higher level of relative risk aversion (i.e. a lower  $\gamma$ ) than men, as discussed above, and the utility from the quality and quantity of children is higher for women (and infants benefit from their parents' leisure). In other words, the marginal utility

from consumption is lower for women and therefore they require larger incentives to work outside the home.

Wages and job-offer rates are taken as exogenous here and we compare employment outcomes when social norms based on a Modern game maximize employment rates of married women. Obviously, in equilibrium the change in labor supply will affect wages and job-offer rates. However, since we expect that preferences for leisure, consumption and household amenities differ between men and women, we would also expect differences in the distribution of employment outcomes, as is the case in a fully symmetric game like that in Modern households.

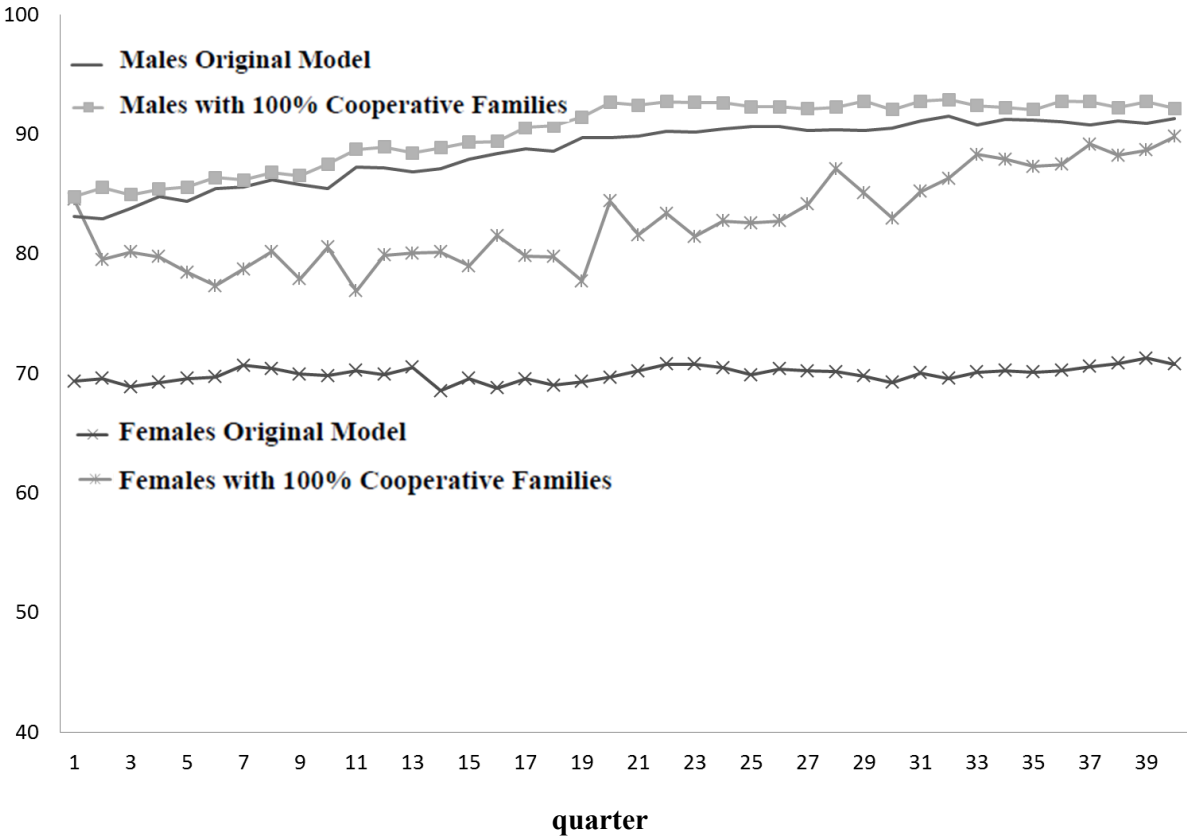
**Figure 7: Simulation 4 - Predicted Employment Rate with 100% Modern Families and Identical Wages and Job Offer Probabilities for Men and Women**



**Simulation 5: All households are Cooperative and employment opportunities for both genders are identical (Figure 8)**

We repeat simulation 4, except that all households are assumed to be Cooperative. As a result of the identical wage and job offer functions, the employment rate of women increases to 0.827 and that of men remains the same as it was in Simulation 3, i.e. 0.9. Thus, in the case of Cooperative households, male and female employment rates differ by 7 percentage points even when facing the same opportunities. The differences in the utility function parameters, i.e. the value of leisure, the level of relative risk aversion and the utility from children, cause women to work less than men. Even though the employment rate is higher for Cooperative women than for Modern women, the gender gap is higher in the Cooperative household since the husband's employment rate remains unchanged.

**Figure 8: Simulation 5 - Predicted Employment Rate with 100% Cooperative Families and Identical Wages and Job Offer Probabilities for Men and Women**



## 6. Concluding Remarks

A dynamic game model is estimated for household labor supply using PSID quarterly data for a sample of married couples who were tracked for up to ten years. The model assumes that the couple plays one of three possible games: a standard game in which the husband is a Stakelberg leader who makes his decisions first and the wife reacts to his outcomes, which we call the Classical household; a Nash game in which husband and wife play a simultaneous symmetric game, which we call a Modern household; and a collective game in which the husband and wife maximize a joint utility function, which we call a Cooperative household. We assume that household type is exogenously determined at the time of marriage as a function of the couple's age and education. The model also assumes dynamic stochastic arrival of children and divorce which affect the couple's lifetime dynamic labor supply.

The estimation results indicate that 57 percent of the 1983-4 cohort of newlywed couples are of the Classical type and the hypothesis that all households are Classical is rejected. The proportion of Modern households is 25 percent and that of Cooperative households is 18 percent. Furthermore, the estimated labor market state outcomes and wages provide a particularly good fit to the data. We find that the labor supply of men is not affected by the type of game while the employment rate for women is lower in Classical households than in Modern households by about 12 percentage points and is higher in Cooperative households than in Modern households by 4 percentage points.

Taking the view that the type of game played in a household is dependent on its socio-demographic characteristics, we find that the Modern and Cooperative households are more likely to be young, better educated and characterized by a higher degree of assortative mating. In other words, the social norms reflected in a Nash symmetric game and in the collective game lead to an increase in the labor supply of women in Modern and Cooperative households while leaving that of their husbands almost unchanged.

The results support the hypothesis that some of the increase in married female labor supply observed in recent decades may be due to changes in social norms that affected the way couples decide on their



joint labor supply. To further investigate this hypothesis will require access to additional data on, for example, couples who married at different points in time in order to determine whether the distribution of households by type changes over time, as claimed here. Moreover, additional specifications of the model, tests of robustness and convincing dynamic games that determine household labor supply are needed to further investigate whether or not changing social norms are an important component in explaining the increase in labor supply of married women.

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## Appendix: The Household Formal Solution

### A: The Classical Household Formal Solution

Let  $V_{ij}^{d_j}(A_{t-j}, \Omega_t, e_{ij})$  be the value function of player  $j$  from a strategy  $d_{ij}$ , as described in section 2.1. The value function is a function of his spouse's outcome  $A_{t-j}$ . Here  $e_{ij}$  is the expected values of  $\varepsilon_i$  that is known to  $j$  in the sub-period of period  $t$ , when  $j$ 's decisions are made. The formal solution is arrived at 5 steps as follows:

#### Step 1:

Since the husband acts first, we need to formulate his expectations of his wife's decisions. To do this, we use the best response function of the wife for each outcome of the husband. The wife can search or not search (OLF). The wife's value function from search is equation (2.11) which is repeated here for convenience:

$$V_{iW}^1(A_{tH}, \Omega_t, e_{tH}) = \{\Pr(\lambda_{iW}) \cdot \max[V_{iW}^E(A_{tH}, \Omega_t, e_{tH}), v_{iW}^{UE}(A_{tH}, \Omega_t, e_{tH})] + (1 - \Pr(\lambda_{iW})) \cdot v_{iW}^{UE}(A_{tH}, \Omega_t, e_{tH})\}$$

where  $\{\Pr(\lambda_{iW})\}$  is the job offer probability (equation 2.7) and  $v_{iW}^E(A_{tH}, \Omega_t, e_{tH}), v_{iW}^{UE}(A_{tH}, \Omega_t, e_{tH})$ , are the value functions of  $E$  and  $UE$ , given the husband's information. The value function of choosing not to search (OLF) is equation (2.12) :  $V_{iW}^0(A_{tH}, \Omega_t, e_{tH}) = v_{iW}^{OLF}(A_{tH}, \Omega_t, e_{tH})$ :

Note that the value function of each outcome is defined by the Bellman equation:

$$(A.1) \quad v_{iW}^k(A_{tH}, \Omega_t, e_{tH}) = E_t \{U_{iW}^k(A_{tH}, \Omega_t, \varepsilon_t) + \beta[V_{iW}^{d_{iW}}(A_{t+1H}, \Omega_{t+1}, \varepsilon_{t+1})] | d_t, \Omega_t, e_{tH}\}, \quad \text{for } k \in \{E, UE, OLF\}$$

where  $\beta$  is the discount factor. The Bellman equation of the wife's DP problem is given by (2.13):

$$V_{iW}^t(A_{tH}, \Omega_t, e_{tH}) = \max_{d_{iW}} \{V_{iW}^{d_{iW}}(A_{tH}, \Omega_t, e_{tH})\}.$$

Since the solution of the DP problem is a function of the husband's outcome  $A_{tH}$ , we evaluate (2.13) for each outcome of the husband, where we assume that the husband only knows  $e_{tH} = (0, 0, \varepsilon_{tH}^3, 0, 0, \varepsilon_{tH}^3)$ . The best response function of the wife is the strategy that maximizes her expected utility for each outcome of the husband, which is given by is (2.14):

$$b_{iW}(A_{tH}, \Omega_t, e_{tH}) = \arg \max_{d_{iW}} \{V_{iW}^{d_{iW}}(A_{tH}, \Omega_t, e_{tH})\},$$

#### Step 2:

Given the above best response function of the wife,  $b_{iW}(A_{iH}, \Omega_t, e_{iH})$ , we can solve for the husband's value from search:

$$(A.2) \quad V_{iH}^1(A_{iW}^e, \Omega_t, e_{iH}) = \left\{ \begin{array}{l} \Pr(\lambda_{iH}) \cdot \max \left[ v_{iH}^E(E(A_{iW} | b_{iW}(\{1,0,0\}, \Omega_t, e_{iW})), \Omega_t, e_{iH}), v_{iH}^{UE}(E(A_{iW} | b_{iW}(\{0,1,0\}, \Omega_t, e_{iW})), \Omega_t, e_{iH}) \right] + \\ (1 - \Pr(\lambda_{iH})) \cdot v_{iH}^{UE}(E(A_{iW} | b_{iW}(\{0,1,0\}, \Omega_t, e_{iW})), \Omega_t, e_{iH}) \end{array} \right\}$$

and his utility from no search:

$$(A.3) \quad V_{iH}^0(A_{iW}^e, \Omega_t, e_{iH}) = v_{iH}^{OLF}(E(A_{iW} | b_{iW}(\{0,0,1\}, \Omega_t, e_{iW})), \Omega_t, e_{iH})$$

where  $A_{iW}^e$  is the wife's expected outcome, given the best response function that defines the wife's optimal strategy for any  $A_{iH}$ . The husband's information regarding the error term is  $e_{iH} = (0, 0, \varepsilon_{iH}^3, 0, 0, \varepsilon_{iW}^3)$ . We can now choose the strategy  $d_{iH} = \{0, 1\}$  that maximizes:  $V_{iH}^i(A_{iW}^e, \Omega_t, e_{iH}) = \max_{d_{iH}} \{V_{iH}^{d_{iH}}(A_{iW}^e, \Omega_t, e_{iH})\}$

The husband's best response function is the strategy that maximizes his expected utility for each expected outcome of his wife (2.16):  $b_{iH}(A_{iW}^e, \Omega_t, e_{iH}) = \arg \max_{d_{iH}} \{V_{iH}^{d_{iH}}(A_{iW}^e, \Omega_t, e_{iH})\}$ ,

If  $d_{iH}^1 = 0$ , then  $a_{iH}^3 = 1$  and we proceed to Step 4 (i.e. the wife's decision). If  $d_{iH}^1 = 1$ , we proceed to Step 3.

### Step 3:

Since the husband decided to search, i.e.  $d_{iH} = 1$ , we randomly draw whether the husband received a job offer according to equation (2.7). If not, then the husband is unemployed ( $a_{iH}^2 = 1$ ) and we proceed to step 4. If the husband did receive a job offer, then the realizations of  $\varepsilon_{iH}^1$ ,  $\varepsilon_{iH}^2$  are revealed. He will accept it ( $a_{iH}^1 = 1$ ) and be employed if  $v_{iH}^E(A_{iW}^e, \Omega_t, e_{iH}) > v_{iH}^{UE}(A_{iW}^e, \Omega_t, e_{iH})$  and reject it and be unemployed ( $a_{iH}^2 = 1$ ) otherwise, where at this point  $e_{iH} = E(\varepsilon_t) = (\varepsilon_{iH}^1, \varepsilon_{iH}^2, \varepsilon_{iH}^3, 0, 0, \varepsilon_{iW}^3)$

### Step 4:

We now solve the wife's employment decision. She already has information regarding her husband's decisions and outcome such that she fully observes  $[\Omega_t, A_{iH}, \varepsilon_{iH}]$ . She also observes  $\varepsilon_{iW}^3$  and therefore

$e_{iW} = (\varepsilon_{iH}^1, \varepsilon_{iH}^2, \varepsilon_{iH}^3, 0, 0, \varepsilon_{iW}^3)$ . We calculate  $b_{iW}(A_{iH}, \Omega_t, e_{iW})$ , and find the strategy  $d_{iW} = \{0, 1\}$  that maximizes:  $V_{iW}^t(A_{iH}, \Omega_t, e_{iW})$ . If  $d_{iW} = 0$ , then  $a_{iW}^3 = 1$  and the game is solved. If  $d_{iW} = 1$ , we proceed to step 5.

**Step 5:**

The wife now searches and we randomly draw whether she receives a job offer according to equation (2.7). If she does not, then she is unemployed ( $a_{iW}^2 = 1$ ) and the game is solved. If she does, then the realizations of  $\varepsilon_{iW}^1, \varepsilon_{iW}^2$  are revealed. She will accept it ( $a_{iW}^1 = 1$ ) and be employed if  $v_{iW}^E(A_{iH}, \Omega_t, e_{iH}) > v_{iW}^{UE}(A_{iH}, \Omega_t, e_{iH})$ , and reject it and be unemployed ( $a_{iW}^2 = 1$ ) otherwise, at this point  $e_{iW} = \varepsilon_t$ .

**B: The Modern Household Formal Solution**

The solution for this game has two steps:

**Step 1**

The husband and wife choose whether or not to search. They act simultaneously with the same state space  $\Omega_t$  and the same information such that  $e_{iW} = e_{iH} = (0, 0, \varepsilon_{iH}^3, 0, 0, \varepsilon_{iW}^3)$ . The utility of each state depends on the strategies of both spouses. Therefore, we calculate the utility for all 2X2 (search or no-search) choices. If the husband does not search, the wife's utility from search is given by equation (2.17), if the husband does search, the wife's utility from search is given by (2.18).

If the husband does search, the wife's utility from no-search is (2.19). If the husband searches, the wife's utility from no-search is (2.20). The value function of the husband  $V_{iH}^1(d_{iW}, \Omega_t, e_{iH}), V_{iH}^0(d_{iW}, \Omega_t, e_{iH})$  has an equivalent format.  $V_{iW}^t(d_{iH}, \Omega_t, e_{iW})$  and  $V_{iH}^t(d_{iW}, \Omega_t, e_{iH})$  are the value functions of the wife's and husband's DP problem, respectively.  $b_{iW}^t(d_{iH}, \Omega_t, e_{iW})$  and  $b_{iH}^t(d_{iW}, \Omega_t, e_{iH})$  are the best response functions of the wife and husband, respectively, as defined in Section 2.2. A Markov Perfect Equilibrium (MPE) of the game is a set of strategies  $d_{iH}^*, d_{iW}^*$  such that  $d_{iW}^* = b_{iW}(d_{iH}^*, \Omega_t, e_{iW})$  and  $d_{iH}^* = b_{iH}(d_{iW}^*, \Omega_t, e_{iH})$ .

**Step 2**

Given the equilibrium  $d_{iH}^*, d_{iW}^*$ , we now calculate the four potential outcomes for the couple:

If  $d_{iH}^* = 0$  and  $d_{iW}^* = 0$ , then  $a_{iH}^3 = 1$  and  $a_{iW}^3 = 1$ .

If  $d_{iH}^* = 1$  and  $d_{iW}^* = 0$  (i.e. only the husband decided to search), we draw a job offer according to equation (2.7). If the husband does not receive a job offer he will be unemployed ( $a_{iH}^2 = 1$ ). If he does, then the realizations of  $\varepsilon_{iH}^1, \varepsilon_{iH}^2$  are revealed. He will accept it ( $a_{iH}^1 = 1$ ) and be employed if  $v_{iH}^1(A_{iW} = 3, \Omega_t, e_{iH}) > v_{iH}^2(A_{iW} = 3, \Omega_t, e_{iH})$  and reject it and be unemployed ( $a_{iH}^2 = 1$ ) otherwise, where at this point  $e_{iW} = e_{iH} = E(\varepsilon_t) = (\varepsilon_{iH}^1, \varepsilon_{iW}^2, \varepsilon_{iH}^3, 0, 0, \varepsilon_{iW}^3)$

If  $d_{iH}^* = 0$  and  $d_{iW}^* = 1$  (i.e. only the wife decides to search), we draw a job offer according to equation (2.7). If she does not receive a job offer she will be unemployed ( $a_{iW}^2 = 1$ ). If she does, then the realizations of  $\varepsilon_{iW}^1, \varepsilon_{iW}^2$  are revealed. She will accept it ( $a_{iW}^1 = 1$ ) and be employed if  $v_{iW}^1(A_{iH} = 3, \Omega_t, e_{iW}) > v_{iW}^2(A_{iH} = 3, \Omega_t, e_{iW})$  and reject it and be unemployed ( $a_{iW}^2 = 1$ ) otherwise, where at this point  $e_{iW} = e_{iH} = E(\varepsilon_t) = (0, 0, \varepsilon_{iH}^3, \varepsilon_{iW}^1, \varepsilon_{iW}^2, \varepsilon_{iW}^3)$

If  $d_{iH}^* = 1$  and  $d_{iW}^* = 1$  (i.e. both decide to search), we draw a job offer according to equation (2.7). If s/he does not receive a job offer s/he will be unemployed ( $a_{ij}^2 = 1$ ). If s/he does receive a job offer, then the realizations of  $\varepsilon_{ij}^1, \varepsilon_{ij}^2$  are revealed. If both receive a job offer, then s/he will accept ( $a_{ij}^1 = 1$ ) and be employed if  $v_{ij}^1(A_{i-j}, \Omega_t, e_{ij}) > v_{ij}^2(A_{i-j}, \Omega_t, e_{ij})$  and reject it and be unemployed ( $a_{ij}^2 = 1$ ) otherwise, where at this point  $e_{iW} = e_{iH} = \varepsilon_t$ .

### ***C: The Cooperative Household Formal Solution***

The solution of this game has two steps:

#### **Step 1**

The husband and wife choose whether or not to search. They act simultaneously with the same state space  $\Omega_t$  and the same information such that  $e_{iW} = e_{iH} = (0, 0, \varepsilon_{iH}^3, 0, 0, \varepsilon_{iW}^3)$ . The utility of each state depends on the strategies of both partners. Therefore, we calculate the utility for all four (search or no-search) choices: both search, neither search, only the wife searches or only the husband searches. The couple chooses the strategy that maximizes their weighted utility.

**The weighted utility when only the wife searches:**

$$(C.1) \quad V_t^{10}(\Omega_t, e_{iW}, e_{iH}) = BP \cdot V_{iW}^1(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 0) + (1 - BP) \cdot V_{iH}^0(d_{iW}, \Omega_t, e_{iH}) | (d_{iW} = 1)$$

where  $V_{iW}^1(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 0)$  is the wife's utility from search when the husband is not searching, defined by (2.17) and  $V_{iH}^0(d_{iW}, \Omega_t, e_{iH}) | (d_{iW} = 1)$  is the husband's utility from no search when the wife is searching. This value function has the symmetric form of the function defined in (2.20).

**The weighted utility when both search:**

$$(C.2) \quad V_t^{11}(\Omega_t, e_{iW}, e_{iH}) = BP \cdot V_{iW}^1(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 1) + (1 - BP) \cdot V_{iH}^1(d_{iW}, \Omega_t, e_{iH}) | (d_{iW} = 1)$$

where  $V_{iW}^1(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 1)$ ,  $V_{iH}^1(d_{iW}, \Omega_t, e_{iH}) | (d_{iW} = 1)$  are defined according to equation (2.18)

**The weighted utility when neither search:**

$$(C.3) \quad V_t^{00}(\Omega_t, e_{iW}, e_{iH}) = BP \cdot V_{iW}^0(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 0) + (1 - BP) \cdot V_{iH}^0(d_{iW}, \Omega_t, e_{iH}) | (d_{iW} = 0)$$

where  $V_{iW}^0(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 0)$ ,  $V_{iH}^0(d_{iW}, \Omega_t, e_{iH}) | (d_{iW} = 0)$  are defined according to equation (2.19)

**The weighted utility when only the husband searches:**

$$(C.4) \quad V_t^{01}(\Omega_t, e_{iW}, e_{iH}) = BP \cdot V_{iW}^0(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 1) + (1 - BP) \cdot V_{iH}^1(d_{iW}, \Omega_t, e_{iH}) | (d_{iW} = 0)$$

where  $V_{iW}^0(d_{iH}, \Omega_t, e_{iW}) | (d_{iH} = 1)$  is defined according to equation (2.20) and  $V_{iH}^1(d_{iW}, \Omega_t, e_{iH}) | (d_{iW} = 0)$  is the husband's utility from search when the wife does not search. This value function has the symmetric form of the function defined in (2.17). We define the solution of this step as a set of strategies  $d_{iH}^*, d_{iW}^*$  that solves the following function:

$$(C.5) \quad \max_{d_{iH}, d_{iW}} \{V_t^{01}(\Omega_t, e_{iW}, e_{iH}), V_t^{11}(\Omega_t, e_{iW}, e_{iH}), V_t^{00}(\Omega_t, e_{iW}, e_{iH}), V_t^{10}(\Omega_t, e_{iW}, e_{iH})\}$$

**Step 2**

Given the set of strategies  $d_{iH}^*, d_{iW}^*$ , Step 2 of this game is identical to Step 2 of the M game, as described in detail in B.

## 7 References

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