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## On the Fit of a Neoclassical Monetary Model in High Inflation: Israel 1972–1990

This paper uses a money-in-the-utility model to show that the Israeli data from the high-inflation and poststabilization period fit well the predictions of a simple, neoclassical framework. Specifically, a single parameter money demand equation that is derived from the model fits the data much better than a Cagan money demand function. The model's implication on the inflation-seigniorage relationship is consistent with the observed fact that while inflation was rising, the inflation-tax revenue remained almost trendless. The labor supply equation derived from the same model is remarkably consistent with the real wage and employment data over the high inflation and stabilization period. There is no evidence for the existence of a "Phillips curve" employment-inflation trade-off. After the stabilization the demand for money seems to shift. This phenomenon may be due to improvements in transaction technologies as well as to expectations for stabilization during the high inflation. We also show that temporary fixed exchange rate-based stabilization and "consumption boom" in the poststabilization period imply low levels of inflation that are consistent with the model.

THIS PAPER USES a money-in-the-utility model (Sidrauski 1969) to show that the Israeli data from the high-inflation (1979–1985) and post-stabilization period (1985–1990) fit well the predictions of a simple, neoclassical framework. Specifically, a single-parameter money demand equation that is derived from the model fits the data much better than a Cagan money demand function. The latter does not fit well low- and high-inflation periods at the same time, and therefore, may erroneously indicate that money demand is not stable. The model's implication on the inflation-seigniorage relationship is consistent with the observed fact that while inflation was rising, the inflation-tax revenue remained almost trendless. This result contradicts the Cagan-type money demand implication of being on the "wrong side of the Laffer curve." The labor supply equation derived from the same model is remarkably consistent with the real wage and employment data over the high inflation and stabilization period. There is no evidence for the existence of a "Phillips curve" employment-inflation trade-off.

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The congruity of the evidence with a neoclassical monetary model suggests that the policy implications of the model are to be taken seriously. In particular, (i) in order to stabilize inflation it suffices to set the expected present value of the government budget deficit (including the central bank) to zero. (ii) There is an immediate welfare gain associated with the stabilization which is estimated to be in excess of 5 percent of GDP in the Israeli case. (iii) There is no evidence to support the view that stabilization is associated with a loss of output (due to increased unemployment) as would be implied by a Phillips curve.

After the stabilization the model overpredicts the demand for money. This phenomenon has been observed for other disinflations and may be due to improvements in transactions technology as well as to expectations for stabilization during the high inflation. Temporary fixed exchange rate-based stabilization, "consumption boom," and low inflation in the poststabilization period are also consistent with the model.

The consistency of the Israeli data with the neoclassical monetary model strengthens the view presented in Sargent's (1986) classical paper, "The End of Four Big Inflations." There Sargent argues that under rational expectations, with credible stabilization programs, a rapid reduction of inflation rates has very small real costs (if any). Some increases in unemployment rates can be traced to the disruption of economic activity during the final phases of the rapid inflation, or to political events, but not to the stabilization *per se*.<sup>1</sup>

The fact that seigniorage is not increasing during the acceleration of inflation is common to all inflationary episodes [see Bental and Eckstein (1990) and the literature cited there]. Sargent and Wallace (1987), among others, argue that this phenomenon can be explained by models in which any given feasible money-financed government deficit is associated with two steady-state inflation rates.<sup>2</sup> This last feature of the theory rationalizes in the opinion of many the need to use "heterodox" policies when stabilizing an inflationary economy; only measures such as wage-price freezes and fixed exchange rates will bring the economy to the low, efficient root.<sup>3</sup>

The possibility of multiple steady states is related to the issue of the stability of the behavioral relationships during the inflationary process and across regimes. An

1. The lack of any major macro disruptions notwithstanding, Sargent notes that "the German inflation was far from 'neutral' and that there were important 'real effects'" (p. 94). Garber (1982) demonstrated that there were significant sectoral shifts in Germany in the wake of the stabilization. In particular, the price distortions during the inflationary period caused labor to flow into the investment goods sector in that period, and into the consumption goods sector after stabilization. Wicker (1986) uses the same evidence for Austria and Hungary that Sargent used in an attempt to argue that Sargent's interpretation of the evidence is wrong. That is, there were severe employment effects of the stabilization programs. However, most of these effects can be traced to reductions in government and financial sector employment. Végh (1992) looks at nine episodes and arrives to similar conclusions.

2. Marcat and Nicolini (1996) show how recurrent episodes of inflation can occur in a multiple-root model with learning.

3. For example, Bruno (1989) claims that: "if . . . the economy was at a high inflation equilibrium, and if, in fact that was a potentially stable equilibrium, then it is no wonder that a fall in the budget deficit, without any accompanying measures that would make the synchronization of nominal magnitudes at a credible low rate of devaluation and wage-price inflation, could shift the economy to an even higher rate of inflation . . ." (p. 294).

important motivation of Cagan's (1956) classical estimation of the demand for money during hyperinflation was to provide evidence in favor of the existence of a stable money demand. However, the extensive empirical work that followed concluded that the demand for money is not stable.

The model here combines Eckstein and Leiderman's (1992) adaptation of money in-the-utility model to the Israeli economy. We extend the model to an open economy with traded and nontraded goods (Drazen and Helpman 1987) and we add the choice of labor supply. Specifically, we have a representative agent model with intertemporal utility maximization. Every period utility is derived from the consumption of an aggregate consumption good, consisting of a traded and nontraded component, real balances, and leisure. The inclusion of real balances in the utility function represents the transactions services provided by money.<sup>4</sup>

We formally demonstrate the following points:

i. A single-parameter standard money demand equation that is derived directly from the neoclassical model fits the data both during the inflation and after the stabilization.<sup>5</sup>

ii. The model fits well the observed seigniorage-inflation relationship during the inflation. In particular, it is consistent with the fact that seigniorage displays hardly any trend while inflation increases dramatically. There is no need to appeal to the multiplicity of stationary equilibria to obtain this result. The fact that seigniorage stayed constant while inflation more than tripled does not provide evidence against the neoclassical theory or against models that possess a unique stationary equilibrium.<sup>6</sup>

iii. The data fit very well the labor supply relationship implied by the model across regimes. In particular, there is no evidence for any inflation-employment trade-off. As can be seen, the turbulence of the inflation rate depicted in Figure 1 is not reflected in the behavior of the labor supply.<sup>7</sup> Figure 3 depicts the behavior of the labor supply which is measured as the share of total available time that an average household spends working. These observations imply that high inflations and their stabilization do not have real output effects.<sup>8</sup>

iv. Using the estimated parameters of the utility function of the representative agent, we provide an assessment of the welfare loss associated with the Israeli infla-

4. It is well known by now that all models that include transactions services of money (for example, McCallum 1990) are equivalent to the Sidrauski model of money and growth.

5. The poststabilization velocity remains "too high" (see below).

6. Eckstein and Leiderman (1992) have demonstrated this point, as well as the fact that the Cagan specification does not fit well the inflation-seigniorage association in the data. Specifically, the Cagan specification implies unbounded velocities as inflation increases. The data indicates that these velocities remain bounded (see Figure 6). Related to this, the Cagan specification implies an inflation-tax "Laff curve," while the data displays no such relationship (see Figure 2).

7. Other measures of the labor supply such as labor force participation, employment rate per household, or unemployment rate also show no evidence for the existence of low frequency trade-offs between inflation and these measures.

8. It is well documented by Bruno (1993) that periods of high inflation are typically characterized by low per capita growth rates. Garber (1982) and Wicker (1986) report on the growth of the banking sector during high-inflation periods and its contraction after stabilization. Israel witnessed a similar development. See also Melnik (1995) and Aiyagari, Braun, and Eckstein (1995) for further discussion.

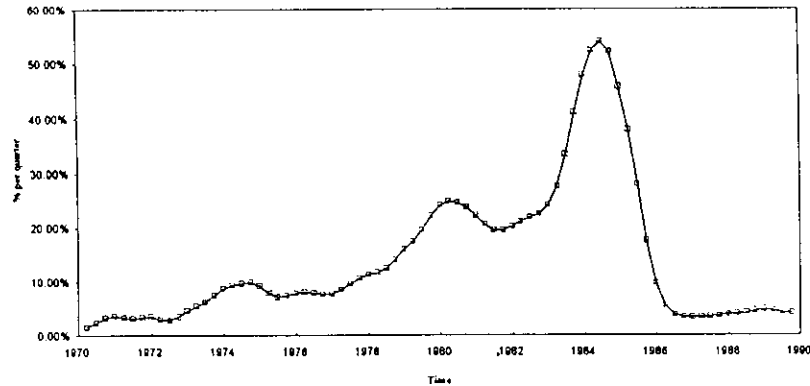


FIG. 1. Quarterly Inflation

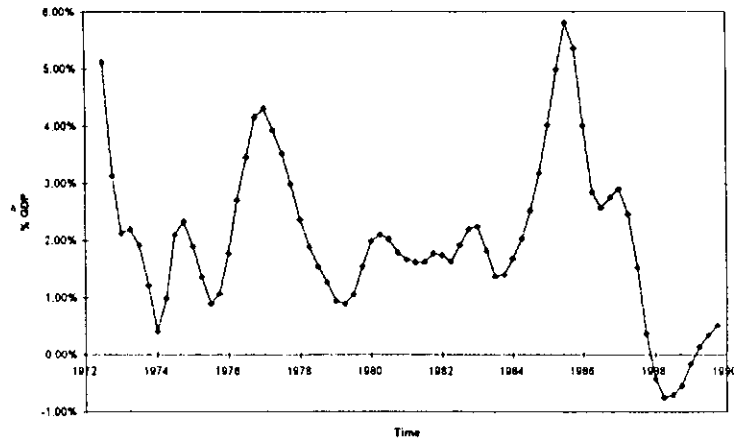
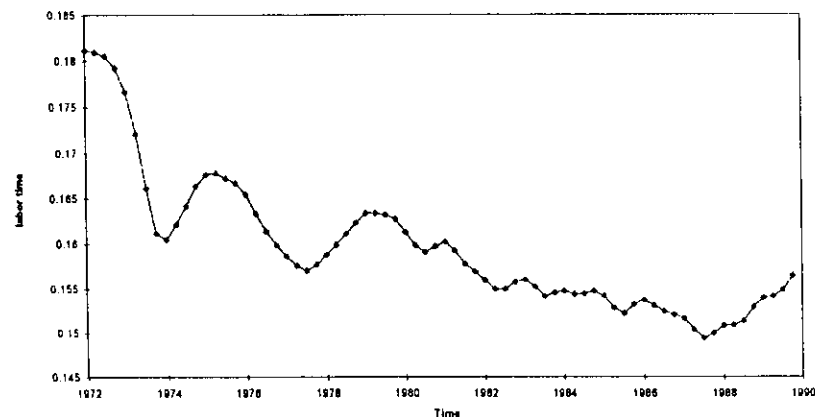


FIG. 2. Actual Seigniorage



tion. At an annual inflation rate of about 20 percent the loss is equivalent to about percent of GDP, while at the peak of more than 400 percent (annually) the cost reaches about 10 percent in terms of GDP.<sup>9</sup>

Despite the remarkable success of the simple model, there are many details that require refinements. One such refinement concerns the poststabilization velocity. Similar to what has been observed after stabilization in other episodes, the demand for real balances after the Israeli stabilization seems to fall short of its preinflationary level (at identical inflation rates). This phenomenon may be due to a positive probability assigned by agents throughout the inflation to the possibility that stabilization will occur.

In addition, we show how the model can be adjusted to consider the common observed attempt to maintain low inflation rates by controlling the rate of nominal devaluation. Taking the actual poststabilization growth rate of GDP and the "consumption boom," as well as the rate of money expansion and the rate of devaluation as given, we show that the model predicts an inflation rate that is close to the persistent poststabilization annual inflation rate of 18 percent. In this way we confirm that the money demand implied by the model fits well the poststabilization data.

The rest of the paper is organized as follows. The next section provides a brief review of the Israeli inflation story. The model is presented in section 2 and empirical implications on money demand and labor supply are presented in section 3. Section 4 looks at the implication regarding seigniorage and section 5 at the welfare implications. In section 6 we analyze the role of expectations regarding stabilization in explaining the poststabilization "shift" of the money demand. Section 7 analyzes the exchange rate stabilization period and section 8 concludes.

## 1. THE ISRAELI INFLATION: A BRIEF HISTORY

The story of the Israeli inflation episode has been described by many (for example, Bruno 1989), and here we provide just a broad outline.

During the 1970s inflation increased from the previous levels of less than 10 percent annually to double digit levels. The first significant acceleration occurred in 1978, following a major liberalization of capital controls and the institution of foreign exchange indexed accounts (the PATAMs). These developments, which were not accompanied by any fiscal steps, brought about a major asset substitution away from unindexed assets. Thus, the inflation tax base eroded, and in 1979 inflation rose to about 5 percent on a monthly basis without any significant change in seigniorage revenues, which remained at about 2 percent of GDP. Inflation continued its upward trend during the first half of the 1980s—it was about 7 percent monthly from the second half of 1981 to the second half of 1983, then rose to an average of about 14 percent till the beginning of 1985, and at the final stages (June 1985) peaked at 30 percent per month (see Figure 1). Seigniorage has an upward trend

9. These costs are due purely to the reduction in real balances. Since the inclusion of real balances, the utility function is an indirect measure of the value added of the transactions services provided by money, the large welfare loss represents some real inefficiencies that are created by inflation.

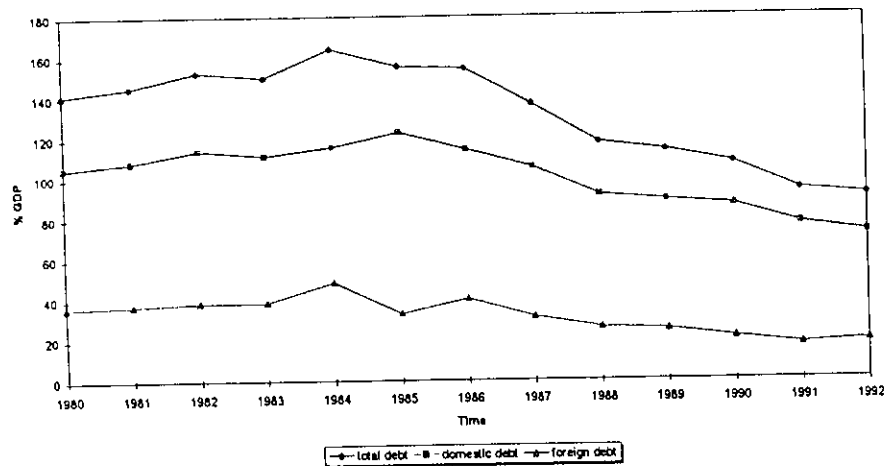


FIG. 4. Public Debt

only toward the end of the episode (Figure 2). The size of the primary deficit (government expenses net of interest payments minus tax revenues) was constantly rising throughout the period. This can be inferred from the development of the internal and the external debt out of GDP (Figure 4).

There were several attempts to deal with the crisis prior to the stabilization program of July 1985. These attempts took the form of "package deals" between the federation of labor unions (the "Histadrut"), the organization of the industrialists, and the government. The first two parties agreed to mutually refrain from increasing wages and prices and the government "contributed" by slowing the devaluation rate and refraining from increasing the prices of subsidized goods (mainly basic food and transportation). All such attempts failed, as they were not accompanied by any corrective fiscal policies.

In July of 1985 a comprehensive government program was enacted. This program was a heterodox one. In addition to cutting the budget deficit to basically zero (mainly by increasing taxes and cutting subsidies, but also by some cuts in government consumption), the government ordered a freeze on prices of key commodities as well as on wages, and fixed the nominal exchange rate. The United States provided a special aid package of 1.5 billion dollars to help stabilize the economy.

Following this program, inflation immediately decreased to 2.1 percent, 1.4 percent, and 1 percent per month, and it stayed at about the latter levels until 1992. The national debt (both internal and foreign) decreased sharply, and the primary deficit was (and still is) maintained at about zero. Remonetization took place following the stabilization of the economy, but on the real side there was no adverse reaction. As a matter of fact, the two years following stabilization are characterized by a boom in economic activity and consumption. Correspondingly, the labor market during this

poststabilization phase shows no sign of slackening. There is a slowdown following these two years, but its relation to the stabilization program is unclear.

The Israeli episode of high inflation could easily fit into Sargent's interpretation of the four major European inflations. All of the main features present in the European episodes are also present in the Israeli one. The acceleration of the inflation as well as the characteristics of the stabilization programs are essentially the same [for details see Bental and Eckstein (1990)]. Only data constraints of the European economies prevent a formal analysis of the neo-classical monetary growth model in those cases along the lines developed below.

## 2. THE REPRESENTATIVE HOUSEHOLD

We view the Israeli economy as being populated by a representative household. Every period  $t$  the household is endowed with one unit of time. It consumes every period  $c_{Tt}$  units of a traded good, and  $C_{Nt}$  of a nontraded good, which we assume to be regarded as perfect substitutes (to simplify aggregation). In addition, the household derives utility from owning real balances.<sup>10</sup> The value of the real balances is the ratio of the nominal balances held by the household,  $M_t$ , and the price index  $Q_t$ .<sup>11</sup> The household allocates a proportion  $\ell_t$  of its time to labor market activities, and enjoys the remainder as leisure. The household is assumed to maximize the expected value of the discounted utility stream, which is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{Tt} + e_t c_{Nt}, M_t/Q_t, (1 - \ell_t)),$$

where  $\beta < 1$  is the subjective discount factor and  $e_t > 0$  is an exogenous variable that measures the subjective rate of substitution between traded and nontraded goods.<sup>11</sup>  $E_0$  is the expectations operator, conditional on all information available at time 0.

In addition to the time endowment that the household receives every period, it is endowed initially with  $M_0$  nominal units of money and  $b_0$  units of interest-bearing government debt. This debt is fully linked to the price level  $Q$ , and bears an interest rate of  $r_t$  between period  $t$  and  $t + 1$ . Accordingly, every period  $t$  the household faces the following budget constraint (written in terms of the traded good):

$$c_{Tt} + (P_{Nt}/\epsilon_t)c_{Nt} + (M_t/\epsilon_t) + (Q_t b_t)/\epsilon_t = (1 - \tau_t)w_t \ell_t + M_{t-1}/\epsilon_t + Q_t [(1 + r_{t-1})b_{t-1}]/\epsilon_t.$$

10. The inclusion of real balances in the utility function is clearly a simplification. It is easy to construct a model whereby the purchase of consumption goods entails the investment of utility-reducing "effort." If this effort is reduced whenever the purchase of goods is done by money, we obtain our specification as a "reduced form." See also Feenstra (1986) for an elaborate discussion of the equivalence between "money-in-the-utility" formulation and other formulations.

11. Our formulation of the preferences here enables us to use the readily available aggregate consumption in the empirical analysis. In section 7 below we relax the formulation to allow for endogenous substitution between traded and nontraded consumption goods.

Here  $P_{Nt}$  is the price of the nontraded good in terms of the domestic currency, and  $\epsilon_t$  is the price of traded goods in terms of the domestic currency.<sup>12</sup> The household sells its labor services for a wage rate of  $w_t$ , in terms of the traded good, and pays a proportional tax at the rate  $\tau_t$ .

Following Drazen and Helpman (1987), the price index  $Q$  is given by

$$Q_t = Q(\epsilon_t, P_{Nt}) \tag{3}$$

where  $Q(\cdot, \cdot)$  is homogeneous of degree 1. We denote  $q_t = Q_t/\epsilon_t$ . Then, due to the homogeneity of  $Q(\cdot, \cdot)$ , we have  $q_t = Q(1, P_{Nt}/\epsilon_t) \equiv q(p_t)$ , where  $p_t = P_{Nt}/\epsilon_t$ . Notice that  $p_t$  is the price of the nontraded good in terms of the traded good, which is the inverse of the real exchange rate. Let  $m_t = M_t/\epsilon_t$  be the real balances in terms of the traded good. Clearly,  $M_t/Q_t = m_t/q(p_t)$ .

The optimal path chosen by the household must satisfy the following Euler first-order conditions:

$$\beta E_t[(1 + r_t)(Q_{t+1}/Q_t)(\epsilon_t/\epsilon_{t+1})(U_c(t+1)/U_c(t))] - 1 = 0, \text{ (w.r.t. } \beta_t) \tag{4}$$

$$(U_M(t)/U_c(t))/q_t + \beta E_t[(\epsilon_t/\epsilon_{t+1})U_c(t+1)/U_c(t)] - 1 = 0, \text{ (w.r.t. } M_t) \tag{5}$$

$$-(U_l(t)/U_c(t)) + (1 - \tau_t)w_t = 0, \text{ (w.r.t. } \ell_t) \tag{6}$$

The symbol  $U_x(t)$  is the marginal utility of  $x$  at time  $t$ . Equation (4) is the standard optimal intertemporal rule for the allocation of consumption, adjusted for the exchange rate changes. Equation (5) represents the demand for real balances and (6) is the labor supply condition. In addition, if the household is to be indifferent between the traded and nontraded goods, we obtain

$$p_t = e_t \tag{7}$$

which follows immediately from the assumption that the traded and nontraded goods are perfect substitutes. To simplify, we assume that  $e_t$  follows an exogenous path.<sup>13</sup> Equation (7) has very straightforward implications. The real exchange rate changes over time like  $e_t$ . Figure 5 shows that despite considerable variations over the sample period, overall there is no clear trend in  $q_t$ , which is related to  $p_t$ .

The gross inflation rate is measured by  $Q_{t+1}/Q_t$ , and is denoted by

$$\pi_{t+1} = Q_{t+1}/Q_t \tag{8}$$

The nominal interest rate between period  $t - 1$  and period  $t$ ,  $R_{t-1}$ , is given by

$$(1 + R_{t-1}) = (1 + r_{t-1})\pi_t \tag{9}$$

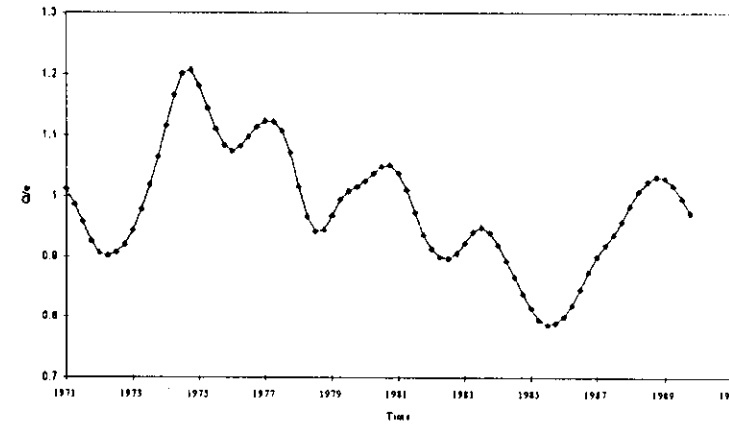


FIG. 5. Normalized CPI ( $Q/t$ )

Then, if  $R_t$  is known at the beginning of period  $t$ , we obtain from (4) and (5):

$$(U_M(t)/U_c(t))/q_t = R_t/(1 + R_t) \tag{10}$$

Equation (10) is the demand for real balances, and underlies the generation of inflation through money creation in the economy.

In order to analyze the empirical implications of the above model we choose the following utility function:

$$U(c_T + ec_N, M/Q, (1 - \ell)) = \{[(c_T + ec_N)^{1-\gamma}(M/Q)^\gamma(1 - \ell)^\alpha]^\mu - 1\}/\theta \tag{11}$$

### 3. EMPIRICAL IMPLICATIONS

In this section we look at the performance of the model with respect to money demand [equation (10)] and labor supply [equation (6)]. These two equations, under the specification (11), imply a particular relationship between the consumption velocity of money and the nominal interest rate, and between labor supply and the real wage. The model predicts that these relationships should hold, regardless of the particular fiscal/monetary regime that prevails as well as the production specification of the economy. This is so because the above Euler conditions should hold in any equilibrium and the empirical analysis is invariant with respect to the underlying structure.<sup>14</sup>

14. We show in an Appendix that the model we developed above can be embedded in a general equilibrium framework. Accordingly, showing that the particular first-order conditions of the consumer problem fit the data is consistent with a broader specification of the economy.

12. This price is represented below by the nominal dollar exchange rate deflated by the U.S. consumer price index.

13. We endogenize the real exchange rate in section 7 below.

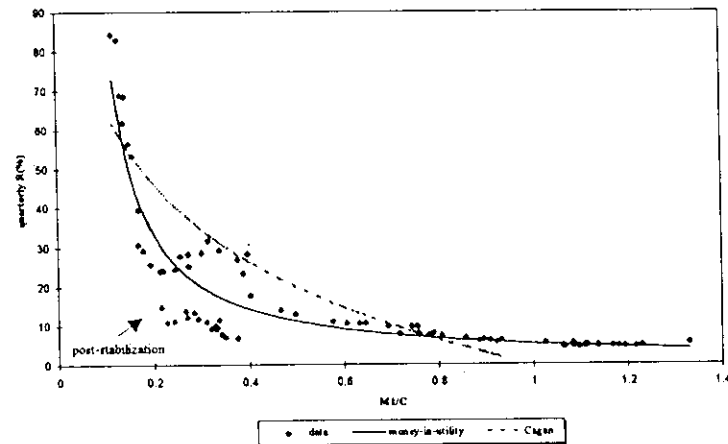


FIG. 6. Money Demand

### 3.1 Money Demand

The parameter  $\gamma$  in the utility function (11) can be interpreted as measuring the transaction requirement of money. A low value of  $\gamma$  implies that money is not essential in acquiring consumption. The higher  $\gamma$  is, the more important the transactions role of money becomes.

According to the specification (11), equation (10) takes the following form:

$$[\gamma/(1 - \gamma)][c_t/m_t] = R_t/(1 + R_t), \quad (12)$$

where  $c_t = c_{Tt} + e_t c_{Nt}$  is aggregate consumption in terms of the traded good.

We rewrite equation (12) as

$$m_t = [\gamma/(1 - \gamma)][(1 + R_t)/R_t]c_t. \quad (13)$$

Equation (12) has a natural interpretation as a standard money demand equation: real balances depend positively on consumption with unit elasticity, and negatively on the nominal interest rate, with changing elasticity.

There are several ways to empirically analyze whether equation (12) fits well the Israeli data from 1970 to 1990. First we graph the actual data points of the nominal interest rate and inverse of the consumption velocity (see Figure 6). This figure shows a remarkable and quite stable relationship between velocity and the nominal interest rate.<sup>15</sup> In Figure 6 we also plot the regression curve implied by (12) (see

below) as well as the regression implied by a Cagan money demand specification the form:

$$m_t/c_t = A \exp(-\alpha R_t). \quad (14)$$

It is quite evident that the specification of our model fits the data much better. This is due to the fact that as inflation and the nominal interest rate increase, a Cagan specification implies that the inverse velocity remains bounded by  $\gamma/(1 - \gamma)$ . In our specification the inverse velocity approaches zero. This implication seems to be counterfactual.<sup>17</sup>

Another simple way is to graph consumption velocity ( $c_t/m_t$ ) and  $R_t/(1 + R_t)$  and see whether they move together as implied by (12). Since (12) represents a low frequency relationship it is reasonable to filter the data, using, say, the Hodrick Prescott filter, to avoid very high frequency fluctuations in the data.

Figure 7 (using  $M_t$  as the measure of money) shows how well equation (12) which is a single parameter money demand equation, performs. In particular, consumption velocity ( $c_t/m_t$ ) follows very closely  $R_t/(1 + R_t)$  as inflation accelerates, and down again as stability sets in.<sup>18</sup> The demand for money is remarkably stable during the entire period up to the stabilization. Estimating the value of  $\gamma/(1 - \gamma)$  by taking the average of  $[R_t/(1 + R_t)]/[c_t/m_t]$ , with  $M_t$  as a measure of money, we obtain 0.050 (0.014) implying a value of  $\gamma$  of 0.048, for the entire sample (1972–1990). For the prestabilization subperiod we obtain value of  $\gamma/(1 - \gamma)$  of 0.055 (0.008) implying values of  $\gamma$  of 0.052. After the stabilization the demand for money seems to have declined: the value of  $\gamma/(1 - \gamma)$  for this period is 0.028 (0.003), implying a value of  $\gamma$  of 0.028 (standard deviations in parentheses).

The poststabilization reduction in the value of  $\gamma$  may be interpreted as indicating a structural decline of the demand for money. Such a shift may be due to the introduction of new payment technologies such as credit cards and ATMs which occurred during the inflationary period. However, as similar shifts have been observed in the European inflations of the early 1920s, other explanations for this phenomenon should also be considered. One such alternative is discussed in section 6.

### 3.2 Labor Supply

The model developed above stipulates that households make optimal decisions regarding their labor-leisure choice [equation (6)]. This assumption implies that behavior in the labor market is determined by the supply of labor: any change in the real wage rate is accommodated by a corresponding change in the amount of time allocated by households to labor market activities. Changes in the monetary regime or rate of inflation bear no relationship to this result. The empirical implication

16. Lucas (1993) has a similar result for the U.S. data.

17. Furthermore, fitting the Cagan money demand to subsamples with low inflation and to subsamples with high inflation would yield very different parameter values. This feature led many to conclude that the demand for money is not stable.

18. Note again the gap that develops after the stabilization.

15. Note the cluster of points that lie a bit closer to the origin. All of these points belong to the post-stabilization era. This is one manifestation of the decline in the money demand. See more on this point below.

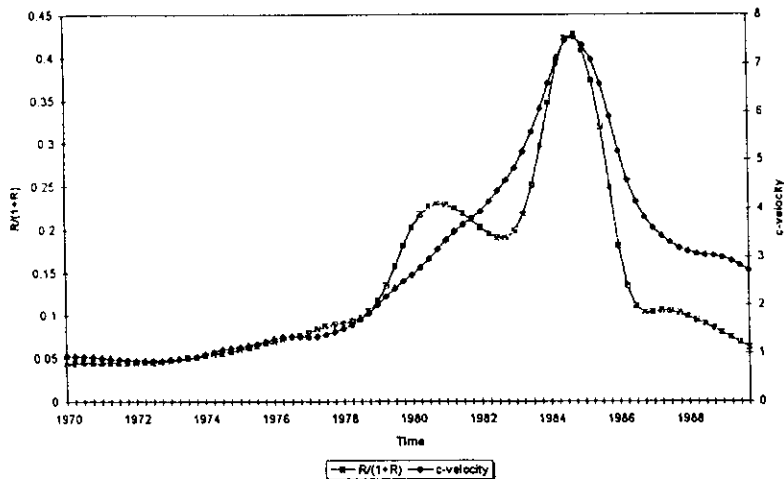


FIG. 7. Consumption Velocity and Interest Rate

this observation is quite clear: the data should not display any stable Phillips curve. There should be no correlation between employment and inflation.<sup>19</sup>

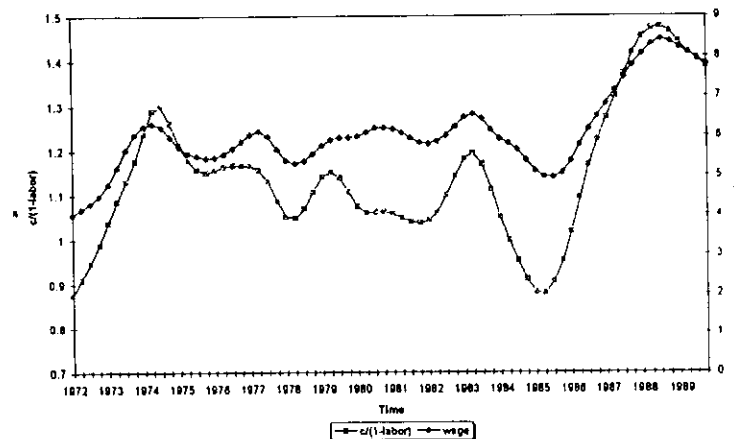
Using the parametric specification (11), equation (6), which describes the labor supply, takes the following form:

$$[(\lambda/(1-\gamma))][c_t/(1-\ell_t)] = (1-\tau_t)w_t. \quad (15)$$

Figure 8 presents the time series of  $c_t/(1-\ell_t)$  and  $w_t$ . The leisure share of time,  $(1-\ell_t)$ , is measured by the proportion of time not used for labor market activities by an average household during quarter  $t$ .<sup>20</sup> It can be clearly seen that  $c_t/(1-\ell_t)$  follows very closely the behavior of  $w_t$  (which is measured by quarterly nominal wages deflated by the price of the traded good in terms of domestic currency). The dramatic changes in the rate of inflation during the period cannot be detected in the figure. In particular, the stabilization of July 1985 and the wage and price freeze that

19. Sargent (1986) argued that the end of the European inflations strongly contradict the prediction of a stable Phillips curve. His evidence is based on limited labor market data that could not confront the hypothesis that labor supply is stable.

20. The exact relationships used for the empirical analysis are the following. The fraction of working time ( $l$ ) is a ratio between two quantities. The numerator is the total of quarterly labor hours. This is computed from the working hours per employee per week (published by the CBS on a quarterly basis) divided by 7 (days per week) multiplied by the number of days per quarter (which we counted) and the quarterly number of employees (published by the CBS). The denominator is the total potential quarterly working hours, which is given by the total quarterly population at the ages 15-65 (published by the CBS) multiplied by 16 (potential working hours per day) multiplied by the number of days per quarter. The quarterly wage rate is another ratio. It is computed as the average monthly wage of an employee (published by the CBS) multiplied by 3 (months per quarter). This is divided by the number of hours worked per quarter given by the working hours per employee per week multiplied by 13 (weeks per quarter), and multiplied by the number of working hours per person per quarter (16 times the number of days per quarter).

FIG. 8. Wages and  $c/(1-labor)$ 

were subsequently imposed during the last two quarters of 1985 leave no mark on the figure. The dramatic increase in the real wage starting in 1985 should have been accompanied, according to a "Phillips curve approach," by a reduction in employment and increased unemployment. Figures 3 and 8 do not provide evidence to support this view.

The value of  $\lambda/(1-\tau)$  can be estimated by the same method that was used earlier for the purpose of estimating  $\gamma$ . In particular, assuming for this purpose that  $\tau$  remain constant, we computed the average of  $w/[c/(1-\ell)]$  over the entire sample period as well as over the prestabilization and poststabilization subperiods. These values are 5.293 (0.396), 5.232 (0.422), and 5.508 (0.165), respectively, with standard deviations reported in parentheses. Using the entire sample average values for  $\gamma$  of 0.048 we obtain that  $\lambda/(1-\tau)$  is 5.56 for the entire period, 5.47 for the prestabilization period and 5.71 for the poststabilization period. We notice a remarkable stability of these estimates, and conclude that the supply of labor remained stable throughout the 1970s and 1980s despite the dramatic monetary events of the period.

### 3.3 Joint GMM Estimation

The estimates reported above were based on simple sample averages using filtered data that emphasize the low frequencies in the data. These estimates are sequential regression results of single equations (one for the money demand and one for the labor supply) each using a single instrument (a constant). Here, we jointly estimate the Euler conditions (4), (5), and (6), for the utility function (11) using Hansen's (1982) GMM.<sup>21</sup> The estimates are based on the unfiltered data, and the instruments are a constant, the growth rate of per household consumption, the

21. Eckstein and Leiderman (1992) have estimated a closely related set of equations with the same data.

growth rate of per household real balances, and one plus the real interest rate.<sup>22</sup> The results (with  $M_1$  as the measure of money) are

$$\beta = 0.997 (0.0131) \quad \gamma = 0.043 (0.001) \quad \theta = -0.412 (0.572)$$

$$\lambda = 5.113 (0.059)$$

The  $J_t$  statistic for the over-identifying restrictions is 26.70, so that the marginal significance level is 0.001. This means that there is more variation in the quarterly (unsmoothed) data than the model can explain.<sup>23</sup>

As can be seen, the main parameters ( $\gamma$  and  $\lambda$ ) are remarkably similar to the estimated values reported above. Furthermore, their standard errors are extremely small. These results imply that the GMM procedure "picked" the constant instrument as the most important one. As for the remaining parameters, the point estimate of the discount rate is quite high, but we cannot reject the hypothesis that the rate of time preference is 1 percent per quarter. The parameter  $\theta$  is not precisely estimated, and we cannot reject the logarithmic specification of preferences ( $\theta = 0$ ).

4. SEIGNIORAGE IMPLICATIONS

As mentioned above, economists found it hard to reconcile increasing inflation rates with trendless (and relatively small) seigniorage levels. Here, we investigate how well our model tracks the seigniorage collected by the Israeli government during the inflationary period and after stabilization. Since the seigniorage data were not used in the estimation procedure above, this investigation may be regarded as a test of the model.

The Israeli government is assumed to consume each quarter (in per household terms)  $g_t$  units in terms of the traded good. Some of this consumption is of non-traded goods, and we assume (to simplify aggregation) that the amount of the non-traded good the government consumes is a fixed proportion of its total consumption. The government finances its expenditures by taxing labor income at the rate  $\tau_t$ , borrowing at home  $(Qb_t)/\epsilon_t$ , by issuing fully indexed debt, borrowing abroad  $\phi_t$ , and by money creation. The government has to serve its debt at home and abroad. The foreign interest rate,  $r_t^*$ , is given. Only the government has access to the foreign capital market, and therefore the foreign interest rate and the domestic one need not equalize.<sup>24</sup> Accordingly, the government (per household) budget constraint (in terms of the traded good) is

$$g_t + Q_t(1 + r_{t-1})b_{t-1}/\epsilon_t + (1 + r_t^*)\phi_{t-1} = \tau_t w_t \ell_t + (Qb_t)/\epsilon_t + (M_{0t} - M_{0t-1})/\epsilon_t + \phi_t \quad (16)$$

22. We implicitly assume here that the nominal interest rate ( $R_t$ ) and the wage rate ( $w_t$ ) are not known with certainty at period  $t$ .

23. In a closed-economy version of the model (where the variation of the exchange rate is not included) the estimated parameters are very close to the ones reported above, but the model is not rejected.

24. This is a realistic feature of the model. The private sector was severely restricted in its access to foreign capital markets throughout the period which we examined. The capital controls are in the process of being liberalized.

We assume that the Israeli government has the choice between two budgetary policies. In one the government finances about 2-3 percent of GDP of its expenditures by seigniorage (defined by  $(M_{0t} - M_{0t-1})/\epsilon_t$ ). In the other budgetary policy the government raises no seigniorage through money creation.

From the beginning of our sample and until July 1985 the government has chosen to be in the first budgetary regime. Figure 2 shows the seigniorage raised by money creation throughout the period as percentage out of GDP. As can be seen, there is very moderate trend in this series until the end of the inflationary period, where seigniorage increases. Eckstein and Leiderman (1992) show in a steady-state analysis that a very slight increase in seigniorage suffices to increase the steady-state inflation rate quite dramatically. This result exists in our model as well, since the money demand is the same as theirs.

For the purpose of this section, we estimate the demand for money using  $M_0$  rather than  $M_1$ . The estimated value of  $\gamma$  is 0.042. We compute the predicted seigniorage as percentage of GDP:

$$\hat{s}_t/y_t = \{(\hat{M}_t - \hat{M}_{t-1})/\epsilon_t\}/y_t \quad (17)$$

where  $\hat{s}$  denotes the estimated seigniorage, and  $\hat{M}$  denotes the estimated demand for the monetary base using equation (13). The actual seigniorage is computed by using the money base from the data. The values of  $y$ ,  $c$ ,  $R$ , and  $Q$  are taken from the data. Figure 9 displays the actual seigniorage and the out-of-steady-state evaluation of the seigniorage obtained by equation (16). The result indicates a good fit between the actual and the model's prediction of the seigniorage both in terms of levels, magnitude of fluctuations, and the moderate trend until 1987. Since in estimating  $\gamma$  we did

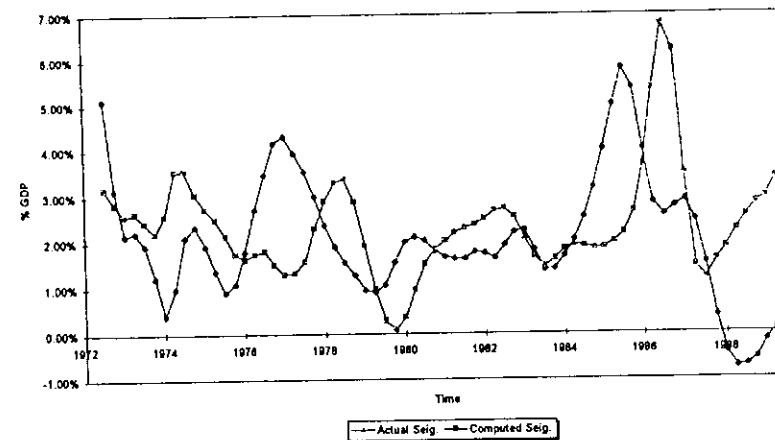


Fig. 9. Actual and Computed Seigniorage



not use the seigniorage data in any direct way, the comparison of the actual and the predicted seigniorage provides an additional input for judging the validity of the model. The large gap between the actual and the computed seigniorage afterward can be explained by the dramatic decrease of the reserve requirements (from about 60 percent in 1985 to 15 percent in 1987), which causes the model to severely overpredict the demand for base money.

##### 5. WELFARE IMPLICATIONS

Explicit models like ours facilitate a welfare analysis of the inflation process. More precisely, we can use such models to assess the period-by-period welfare loss due to inflation.

Consider the household's utility function given by (1). The Bellman equation associated with the optimal plan states that the maximal utility the household obtains can be broken down into the contribution of the current period to that utility and the discounted expected future contributions to that utility.

We want to compare the maximal utility the household obtains under the observed policy of the Israeli government to a potential utility that would have been obtained if the government were to cut expenditures and stop inflation.

Suppose that the change in policy is neutral with respect to private consumption, real wages, and the ratio of the price index ( $Q$ ) to the price of traded goods in terms of domestic currency ( $\epsilon$ ). Under these circumstances, the expected discounted value of future utility must be lower under the observed (inflationary) policy. As a result, the difference between the maximal potential single-period utility and maximal single-period utility under the observed inflation underestimates the welfare gain obtained from stopping the inflation.

We compute two measures of utility, both with  $\gamma = 0.04$ , and  $\lambda = 5$ .<sup>25</sup> One is the predicted utility. This utility is computed using the data on consumption and the nominal interest rate in equation (12) to predict the real balances. Using the actual consumption and leisure and the predicted real balances, we compute the predicted utility. The second measure is the potential utility. It is obtained by computing the money demand from equation (12), using the actual consumption data, and a real interest rate of 1 percent per quarter. Figure 10 displays the single period predicted and potential utilities. As can be clearly seen, the potential utility is higher.<sup>26</sup>

We next compute a measure of the welfare loss due to inflation. Our measure is based on the compensation in income that is needed to make the representative household indifferent between the predicted utility it obtains under the inflationary regime and the potential utility it would have obtained if inflation were to stop.<sup>27</sup> In both cases, we use consumption and leisure as given by the data.

As can be seen from Figure 11, the required compensation reaches a peak of

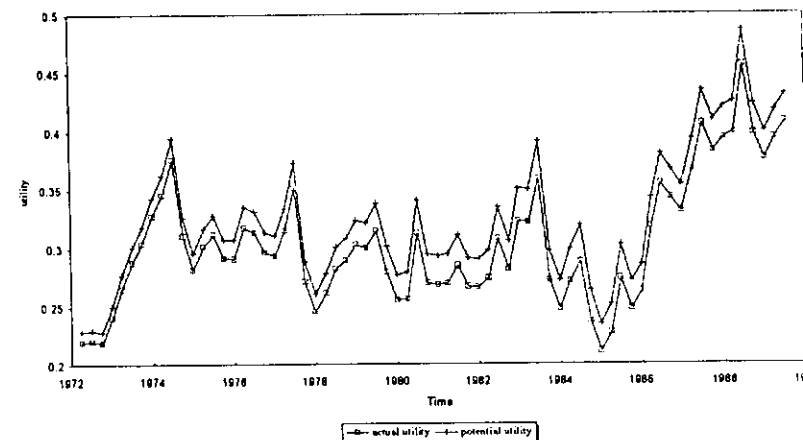


FIG. 10. Actual and Potential Utility

about 8 percent of GDP at the end of the inflationary episode, and falls but remains positive even after stabilization. It is important to note that the welfare cost of inflation increases substantially as inflation increases. The equivalent utility loss of 8 percent of GDP explains the urgency the government eventually felt to stabilize the economy.<sup>28</sup>

The welfare cost after stabilization stays high relative to the cost incurred for the same inflation rates at the early 70s because the Bank of Israel held the nominal interest rate at high levels, thus reducing the demand for money. Using the actual data in the calculations, the poststabilization welfare loss is even higher. This is due to the fact that real balances did not attain their preinflationary levels.

As noted above, this gap between real balances before and after inflationary periods (for the same inflation rates) is a common phenomenon. Clearly, using the lower poststabilization money demand and a value of  $\gamma$  of 0.028 in the welfare calculations would reduce the estimated welfare loss due to inflation.<sup>29</sup>

##### 6. MONEY DEMAND WITH EXPECTED STABILIZATION

To provide a possible explanation for the seeming reduction in the money demand after stabilization we follow the work of Flood and Garber (1980), LaHaye (1985), and Drazen and Helpman (1987). These papers studied the shift of the demand for money that occurs during an inflationary process in relation to the expectation

28. The stopping of inflation was the only economic goal on the agenda of the "national unity government" formed at the end of 1984.

29. It should be noted that even at the lower value of  $\gamma$ , the welfare loss due to inflation obtained from our model is substantially higher than that found by models which are based on U.S. data at comparable inflation rates (for example, Lucas 1993).

25. The value of  $\theta$  does not play any role in this computation.

26. A similar picture is obtained when actual real balances are used instead of the predicted ones.

27. The equivalence in terms of income is obtained by using the fact that consumption is 60 percent out of GDP on average.

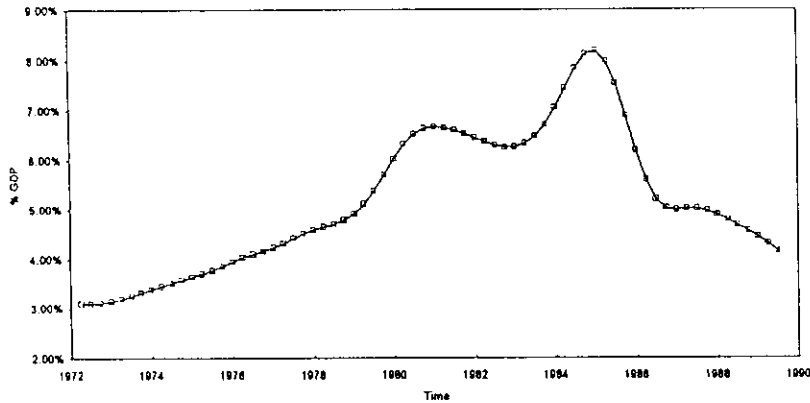


FIG. 11. Compensation (% GDP)

agents form with respect to a stabilization. Here we suggest a simple way to empirically analyze this approach in order to explain the reduction in the estimated value of  $\gamma$  in the poststabilization period.

As mentioned above, an alternative explanation for this seeming decline in the demand for money is the growth in transactions services provided by the banking sector [in the Israeli case there was a big increase in the use of credit cards, automated tellers, etc.; see Melnik (1995)]. However, these changes took place during the inflation, and therefore should have been manifested in the demand for money before the stabilization.<sup>30</sup>

We assume that throughout the high-inflation period households assign a positive probability to the event that the inflation will stop. Clearly, if this event occurs, the return on money will increase at once. Accordingly, the demand for real balances during the inflation reflects the probability assigned to the possibility that the inflation will stop within the next period. Therefore, demand for money is "too high" (and the velocity "too low") relative to the observed inflation rate that materializes when stabilization fails to occur. Estimating money demand parameters from the raw data during the inflationary period, as done above, is therefore likely to yield parameter values that bias the estimated demand upward.

In principle an estimation procedure could be designed that would jointly estimate the parameters of the money demand equation as well as the probabilities assigned to regime changes (both from the inflationary regime to a stable environment and vice versa). However, since we observe only one switch in the regime (from inflation to stability), such a procedure cannot be implemented. Accordingly, we

30. Sargent (1986) explains an analogous phenomenon (the substantial poststabilization increase in central bank notes and demand deposits) by pointing out that after the stabilization the central bank issued bank notes in exchange for real bills, rather than government debt.

assume that once the economy is stabilized, it does not revert to an inflation.<sup>31</sup> This means that the "true" money demand can be estimated from the poststabilization data. Using the estimated parameters, we use the prestabilization data to estimate the probability of a regime switch.

Specifically, we assume that at every period  $t$  the economy may be in one of two regimes. Regime 0 is one in which inflation is maintained at a constant level, so that  $\pi_{t+1} = \pi_0 \geq 1$ . Whenever the change in prices is stabilized, so is the change in the exchange rate. Denoting the change in the price of traded goods in terms of the domestic currency by  $\eta_{t+1}$ , this assumption implies that after stabilization  $\eta_{t+1} = \eta_0 \geq 1$ . Alternatively, regime 1 may prevail, for which  $\pi_{t+1}$  and  $\eta_{t+1}$  may be changing (increasing). Households know which of the two regimes prevails at the beginning of period  $t$ . They do not know whether the regime change will occur during the prevailing period.<sup>32</sup> However, conditional on the regime, households are assumed to know what the price changes and devaluation will be. We assume that the inflationary regime will change during period  $t$  with probability  $\varphi_t$ .

The labor-leisure decision, as implied by equation (6), is independent of the probability of a change in regime. However, the demand for money as given by (5) does depend on the expected future regime. Given our specification of preferences, equation (5) becomes

$$\begin{aligned} & [\gamma/(1-\gamma)](c_t/m_t) + \beta E_t \{ (1-\varphi_t)(1/\eta_{t+1})(1/\hat{c}_{t+1}) \\ & + \varphi_t(1/\eta_0)(1/\hat{c}_0) \} - 1 = 0 \end{aligned} \quad (18)$$

where  $\hat{c}_{t+1}$  denotes the consumption growth  $c_{t+1}/c_t$  if the inflationary regime continues to prevail, and  $\hat{c}_0$  otherwise.

We simplify by assuming that the consumption growth rate is independent of the regime change. In this case, equation (18) can be used to obtain the probability as associated by the households to the event that the inflationary regime will change a period  $t$ :

$$\varphi_t = \frac{1 - [\gamma/(1-\gamma)](c_t/m_t) - \beta(1/\hat{c}_{t+1})(1/\eta_{t+1})}{\beta(1/\hat{c}_{t+1})(1/\eta_0) - (1/\eta_{t+1})} \quad (19)$$

Thus, if the possibility of a regime change is ignored, using an estimated value of  $[\gamma/(1-\gamma)]$  which is obtained from a money demand equation like (5) will overestimate the demand for money during an inflationary period. Wrongly assuming that the inflationary regime continues with probability one, the demand for money should be low, reflecting the high nominal interest rates associated with the inflation. However, households who expect (with some positive probability) a regime change, will hold larger real balances in accordance with that probability.

To estimate an average probability of a regime change during the inflationary pe-

31. With this assumption the model is nonstationary. However, a very small probability that the economy may revert from a stable path to the inflationary one is sufficient to support stationarity. Accordingly, our case should be regarded as a limiting case of stationary environments.

32. Ruge-Murcia (1995) estimates a model of the Israeli inflation in which agents infer from the observed data whether a regime change has taken place. We assume that once such a change has taken place, it is immediately recognized by everyone.

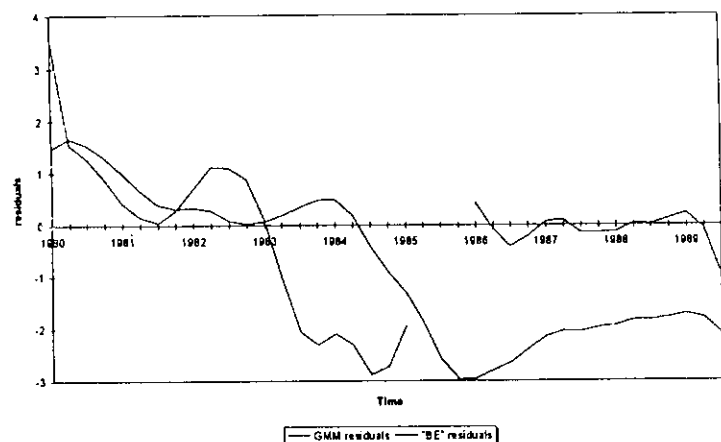


FIG. 12. Residuals

riod, we substitute the estimated value of  $\gamma/(1 - \gamma)$  from the poststabilization data into (19), use the observed values of the velocity ( $c_t/m_t$ ), the consumption growth  $\hat{c}_{t+1}$  and the change of the price of traded goods in terms of the domestic currency  $\eta_{t+1}$ . The poststabilization value of the latter,  $\eta_0$ , is the average change of the price of traded goods in terms of the domestic currency after stabilization has occurred. The average probability assigned to a regime change in the coming quarter,  $\phi$ , obtained in this way, is 0.75. This is a high probability that reflects the relatively large reduction in the demand for money after the stabilization. However, this value constitutes an upper bound, as we assigned a probability of zero to the event that after stabilization there could be a return to the inflationary regime. Clearly, if households assign a positive probability to the resumption of inflation, then the estimated value of  $\gamma$  after the stabilization underestimates its true value.<sup>33</sup>

One way to illustrate the correction achieved by this extension to the money demand equation is to compare the residuals obtained from the parameters estimated using the GMM procedure over the entire sample period which ignores the probability of a regime change to the residuals obtained by the procedure described above. Define the GMM residuals by

$$u_{GMM} = 1/[\Gamma_{GMM}][1 - \beta(1/\eta_{1+t})(1/\hat{c}_{t+1})] - (c_t/m_t) \quad (20)$$

where  $\Gamma_{GMM} = \gamma_{GMM}/(1 - \gamma_{GMM})$ , and

$$u_{BE} = 1/[\Gamma_{PS}][1 - \beta(1/\hat{c}_{t+1})\{\phi(1/\eta_{1+t}) + (1 - \phi)\}] - (c_t/m_t) \quad (21)$$

where  $\Gamma_{PS} = \gamma_{PS}/(1 - \gamma_{PS})$  stands for the poststabilization parameter values. As can be seen from Figure 12, the GMM residuals have a clear trend, where all post-

stabilization residuals are negative. The residuals of the alternative procedure (denoted "BE" residuals) are basically trendless. They become large and negative about two years prior to the stabilization, reflecting the fact that the probability assigned to a regime change as implied by the model should fall toward stabilization.

## 7. POSTSTABILIZATION INFLATION

After stabilization, inflation settled on an annual rate of 18 percent. During the same period the average annual nominal devaluation amounted to 7 percent with no devaluation at the initial period. In addition, aggregate consumption grew by an annual rate of 7.4 percent, while GDP grew, on average, at 3.8 percent.<sup>34</sup> Clearly, the large deviation of the rate of nominal devaluation from inflation could not persist forever and, in fact, the gap has been closed during the early 1990s. Still, the question is whether as a temporary phenomenon these observations are consistent with our model. Here, we show that if we take the paths of the exchange rate devaluation, monetary expansion, and output growth as exogenous, the observed inflation rate is consistent with the model's predictions.

The formulation above in which the traded and nontraded goods are perfect substitutes is not well suited to discuss the relationship between nominal exchange rate changes and inflation. In particular, an equilibrium in which both traded and nontraded goods are consumed requires  $e_t = P_N/\epsilon_t$ , where  $e_t$  is the exogenous marginal rate of substitution between both types of consumption goods. Clearly, any small deviation from this condition means that either of the two types of goods will not be consumed. To avoid this extreme implication, we relax the assumption that the marginal rate of substitution between traded and nontraded goods is independent of prices.

For this purpose we omit the labor choice, and amend the periodic preferences to be  $U(c_T, c_N, M/Q)$ . Under these circumstances the Euler first-order conditions become

$$(1 + r_t)\beta E_t(\pi_{t+1}/\eta_{t+1})(U_{cT}(t+1)/U_{cT}(t)) - 1 = 0, \quad (22)$$

and

$$\epsilon_t(U_M(t)/U_{cT}(t)) + \beta E_t(1/\eta_{t+1})(U_{cT}(t+1)/U_{cT}(t)) - 1 = 0. \quad (23)$$

The new condition pertains to the static choice between traded and nontraded goods:

$$(\epsilon_t/p_N)(U_{cN}(t)/U_{cT}(t)) - 1 = 0. \quad (24)$$

To facilitate the comparison between this specification and the previous one, we specify

34. This poststabilization "consumption boom" is common to many economies that stabilized their currency. It may be explained as a result of a wealth effect (see, for example, Rebelo and Végh 1995).

33. The high estimated value of  $\phi$  would be lower if we allow the value of  $\gamma$  to decrease during the high-inflation period.

$$U(c_{Tt}, c_{Nt}, M_t/Q_t) = \{[(c_{Tt})^\nu (c_{Nt})^{1-\nu}]^{1-\gamma} (M_t/Q_t)^\gamma\}^\theta - 1/\theta. \quad (25)$$

In addition, we specify for the price index:

$$Q_t = (\epsilon_t)^\alpha (p_{Nt})^{(1-\alpha)}. \quad (26)$$

We use these specifications in order to derive a relationship between the growth rate of the economy, the money supply growth, the rate of change of the price of traded goods in terms of the domestic currency, and the rate of inflation.

We use equation (24), which becomes

$$[(1-\nu)/\nu](\epsilon_t/p_{Nt})(c_{Tt}/c_{Nt}) - 1 = 0, \quad (27)$$

together with equation (26) to compute

$$\pi_{t+1} = (\epsilon_{t+1}/\epsilon_t)[(c_{Tt+1}/c_{Tt})(c_{Nt+1}/c_{Nt})]^{(1-\alpha)}. \quad (28)$$

To simplify, we concentrate on a special case in which the (gross) rates of change of all relevant variables is constant. In particular, let the economy grow at a rate  $g$ . We assume that consumption of nontraded goods grows at the same rate. Let the consumption of traded goods grow at a rate  $x$ . Let the money supply grow at  $\mu$ , and let the real interest rate remain constant.

Under these circumstances, the demand for money, derived from equations (22) and (23), and using

$$\begin{aligned} U_{c_T}(t+1)/U_{c_T}(t) = \\ (c_{Tt+1}/c_{Tt})^{\nu(1-\gamma)-1} (c_{Nt+1}/c_{Nt})^{(1-\nu)(1-\gamma)} \\ ((M_{t+1}/Q_{t+1})/(M_t/Q_t))^\gamma, \end{aligned} \quad (29)$$

becomes

$$[\gamma/\nu(1-\gamma)][c_{Tt}/(M_t/\epsilon_t)] = 1 - (1/\eta)[(1/x)(g)]^{(1-\alpha)/(1+r)}. \quad (30)$$

Note that equation (30) is perfectly analogous to (12) when  $\eta = \pi$ , and  $x = g$ . From equation (30) we obtain

$$x = c_{Tt+1}/c_{Tt} = (M_{t+1}/M_t)/(\epsilon_{t+1}/\epsilon_t) = \mu/\eta. \quad (31)$$

Equation (31) implies that consumption of traded goods is likely to grow at a rate that exceeds the economy's growth rate if the growth of the money supply exceeds the rate of change of the price of traded goods.

Substituting equation (31) into (28), and using our assumptions about the growth rates, we get that the inflation rate is

$$\pi = \eta^\alpha (\mu/g)^{(1-\alpha)}. \quad (32)$$

Equation (32) implies that any expansion of the money supply at a rate that exceeds the economy's growth rate ( $\mu > g$ ), while holding the price of traded goods in terms of domestic currency fixed ( $\eta = 1$ ), generates inflation. However, if the nominal exchange rate is maintained at a fixed level, then  $\eta = \pi^*$ , where  $\pi^*$  denotes the gross inflation abroad. In this case, inflation is given by

$$\pi = (\pi^*)^\alpha (\mu/g)^{(1-\alpha)}. \quad (33)$$

In particular, the domestic inflation can be controlled (for a while) by halting the nominal devaluation.

This policy cannot be maintained forever. In the long run, the money supply has to be expanded at the growth rate of the economy (which is also the growth rate of consumption of both the traded and nontraded goods) and the nominal devaluation rate should match the rate of the foreign inflation, to keep the price of traded goods in terms of domestic currency constant. With this combination, the price level remains stable.

As pointed out above, some aspects of the poststabilization behavior of the Israeli economy match this story quite well. On a yearly basis, the (geometrically averaged) gross growth rate of  $M_t$  ( $\mu$ ) for the years 1986-9 was 1.324. The average nominal devaluation against the dollar was 1.069, while prices in the United States rose on average by 1.036, implying a value of  $\eta$  of 1.032. Using these number in equation (31) yields a predicted average growth rate of the consumption of traded goods of 1.28, a number which seems too high. However, this result, when substituted into equation (28), may yield a quite reasonable prediction for the inflation rate, as given by equation (32). To show this, we need to assess the value of  $\alpha$ .

According to calculations of the Bank of Israel, 44.2 percent of the output of the business sector in these years was traded (the 1995 report, Table B-2). The business sector constituted about 65 percent of GDP (Table B-1). This implies that the weight of traded goods is about 29 percent in GDP, assuming that only the business sector produces traded goods.<sup>35</sup> Assuming that the weight of traded and nontraded goods in the CPI matches these calculations, we obtain a value of  $\alpha = 0.29$ . Thus, equation (32) yields an inflation rate of 1.20, which is very close to the actual average yearly inflation in these years of 1.18. In fact, this calculation demonstrates that given the performance of the real side of the economy and the expansion rate of the money supply, the money demand implied by the model fits well the poststabilization data.

## 8. CONCLUSION

We have used a very simple model to tell the story of inflation and stabilization. We used the Israeli case, and describe the behavior of inflation, consumption, real

35. Consumption is likely to be even more biased toward nontraded goods.

balances, employment, exchange rate, wages, and seigniorage. The story told by the model, despite its simplicity, is remarkably close to reality. That is, the comovements of the variables as predicted by the model fit well the main observed features of the data.

There are certain refinements of the basic model that help improve its performance. We show that a modification may be able to account for the seeming decline of the demand for real balances after stabilization. Adding uncertainty about the stabilization date implies that the demand for real balances during the inflation does not decline as much as would be predicted if the inflation were to increase with certainty. Another modification is used to demonstrate that the model is consistent with the poststabilization inflation while the nominal exchange rate remained fixed at a given level. As the fixed exchange rate policy is not sustainable in face of an inflation rate that exceeds that of the trade partners, the success of the model in this case as well as the performance of the simple model before and after the stabilization indicate that the model is not very sensitive to the temporariness of the current regime and to expected regime changes.

The success of the approach lends credence to the welfare and policy implications of the model. Specifically, using the simple model, we measure a substantial welfare loss due to high inflation and no output cost of stopping it. These features may not be consistent with views held by certain economists, but are remarkably consistent with the political popularity of stopping inflation.

#### APPENDIX

In this Appendix we describe a simple general equilibrium model that embeds the household optimization problem of section 2 in the text. By doing so, we show that the model we estimate is consistent with a general equilibrium setting.

#### Production

The economy produces a traded good and a nontraded good. To simplify, we ignore all factors of production except labor.

Aggregate nontraded goods production,  $Y_{Nt}$ , is given by

$$Y_{Nt} = A_{Nt} L_{Nt}^\delta, \quad 0 < \delta < 1 \quad (\text{A1})$$

where  $L_{Nt}$  denotes aggregate labor input into the nontraded good production, and  $A_{Nt}$  follows a stochastic process with growth.

Traded goods production,  $Y_{Tt}$ , is given by

$$Y_{Tt} = A_{Tt} L_{Tt} \quad (\text{A2})$$

where  $L_{Nt}$  denotes aggregate labor input into the nontraded good production, and  $A_{Tt}$  follows a stochastic process with growth (with the same mean as  $A_{Nt}$ ).

#### Profit Maximization

For simplicity assume that the entire sector of the nontraded good consists of a single, price-taking, profit-maximizing firm (clearly, any exogenously given finite number of firms would yield an equivalent characterization). The traded good sector consists of any number of price-taking and profit-maximizing firms. The zero-profit condition in the traded good sector and profit maximization in the nontraded good sector imply

$$W_t = \epsilon_t A_{Tt} \quad (\text{A3})$$

and

$$W_t = P_{Nt} A_{Nt} A_{Nt} \delta L_{Nt}^{\delta-1} \quad (\text{A4})$$

where  $W_t$  denotes the nominal wage, and  $P_{Nt}$  and  $\epsilon_t$  are the prices of the nontraded and traded goods in terms of the domestic currency, as defined in the text.

Equation (A4) implies a nominal profit,  $Z_{Nt}$ , in the nontraded good sector of

$$Z_{Nt} = (1 - \delta) Y_{Nt}. \quad (\text{A5})$$

We also have the definitions from section 2:

$$\begin{aligned} w_t &= \frac{W_t}{\epsilon_t}, \\ p_t &= \frac{P_{Nt}}{\epsilon_t}, \\ q_t &= \frac{Q_t}{\epsilon_t}. \end{aligned} \quad (\text{A6})$$

#### Households

The household problem is as defined by the preferences (1) and budget constraints (2) in section 2. The budget constraint is corrected for the per-household profits denominated in terms of the traded good,  $z_t$ , generated by the nontraded good sector, which are taken as lump-sum:

$$\begin{aligned} c_{Tt} + (P_{Nt}/\epsilon_t)c_{Nt} + (M_t/\epsilon_t) + (Q_t b_t)/\epsilon_t &= (1 - \tau_t)w_t \ell_t + M_{t-1}/\epsilon_t \\ &+ Q_t[(1 + r_{t-1})b_{t-1}]/\epsilon_t + z_t. \end{aligned} \quad (2')$$

#### The Government

Each period, the government consumes  $G_{Tt}$  units of the traded good and  $G_{Nt}$  units of the nontraded good. The government budget constraint is

$$G_{Tt} + \frac{P_{Nt}}{\epsilon_t} G_{Nt} + Q_t(1 + r_{t-1})B_{t-1}/\epsilon_t + (1 + r_t^*)\Phi_{t-1} =$$

$$\tau_t w_t L_t + (Q_t B_t)/\epsilon_t + (\dot{M}_{0t} - \dot{M}_{0t-1})/\epsilon_t + \Phi_t \quad (A7)$$

Equation (A8) is equivalent to equation (16) in section 4, except that equation (A8) is written in aggregate terms, so that  $L_t$  is total employment,  $B_t$  is the stock of CPI indexed bonds,  $\Phi_t$  is the aggregate government foreign debt, and  $\dot{M}_{0t}$  is the aggregate stock of outside money at period  $t$ .

#### Resource Constraints

Total uses of the nontraded good must equal production:

$$C_{Nt} + G_{Nt} = Y_{Nt} \quad (A8)$$

where  $C_{Nt}$  denotes aggregate private consumption of the nontraded good.

The traded good can be imported, so that

$$C_{Tt} + G_{Tt} = Y_{Tt} + IM_t \quad (A9)$$

where  $C_{Tt}$  denotes aggregate private consumption of the traded good and  $IM_t$  is the net import.

The current account deficit is matched by a capital account surplus:

$$IM_t = \Phi_t - (1 + r_t^*)\Phi_{t-1} \quad (A10)$$

The labor market equilibrium requires

$$L_{Nt} + L_{Tt} = L_t \quad (A11)$$

and

$$L_{Nt} + L_{Tt} = N_t \ell_t \quad (A12)$$

where  $N_t$  is the number of households in period  $t$ .

In addition, we require

$$\dot{M}_{0t} = N_t M_t \quad (A13)$$

$$B_t = N_t b_t \quad (A14)$$

$$C_{Tt} = N_t c_{Tt} \quad (A15)$$

$$C_{Nt} = N_t c_{Nt} \quad (A16)$$

and

$$Z_t = N_t z_t \quad (A17)$$

#### Equilibrium

An *equilibrium* is a stochastic path of  $\{c_{Nt}, c_{Tt}, M_t, b_t, \ell_t, Y_{Nt}, Y_{Tt}, L_{Nt}, L_{Tt}, L_t, C_{Nt}, C_{Tt}, W_t, w_t, P_{Nt}, \epsilon_t, p_t, Q_t, q_t, \pi_t, z_t, Z_t, R_t, r_t, IM_t, \Phi_t, \dot{M}_{0t}\}$  given a stochastic path of  $\{A_{Nt}, A_{Tt}, G_{Nt}, G_{Tt}, N_t, e_t, r_t^*, \tau_t, B_t\}$  for  $t = 1, 2, \dots$  and given initial conditions  $\dot{M}_{00}$ ,  $\Phi_0$ , and  $B_0$ , such that the following equations hold:

- (i) Household problem: Budget constraint (2') and Euler equations (4), (5), and (6).
- (ii) Government budget constraint: Equation (A7).
- (iii) Production and profits: (A1), (A2), and (A5).
- (iv) Resource Constraints: Equations (A8), (A9), (A10), and (A11).
- (v) Prices: Equations (3), (7), (8), (9), (A3), (A4), and (A6).
- (vi) Aggregation: (A12), (A13), (A14), (A15), (A16), and (A17).

#### Remark

In general, the equilibrium path generated by this environment is not Pareto optimal. Nevertheless, there are examples in the literature of similar economies in which equilibria have been shown to exist. Therefore we conjecture that under sufficient regularity conditions, an existence proof can be constructed also for this environment.

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## Comment on ON THE FIT OF A NEOCLASSICAL MONETARY MODEL IN HIGH INFLATION: ISRAEL 1972-1990,

by Mark W. Watson

One of the important successes in empirical macroeconomics is the relative stability of money demand during the extremes of hyperinflations. Beginning with the seminal work of Cagan, this stability has been found in many different countries over many sample periods. Bental and Eckstein study the Israel inflation of the 1970s and 80s and persuasively demonstrate this stability in an environment that includes inflation ranging from essentially 0 percent to over 200 percent per annum. Figure 7 in their paper is a beautiful summary of the empirical success of a simple model of money demand, and the paper is worth remembering if only for this remarkable picture.

The paper makes several other contributions, however. It proposes a simple representative agent-optimizing model that rationalizes the demand for money function. It demonstrates that the model does a reasonable job mimicking the trends in money demand, interest rates, seigniorage, and employment. It shows that the disinflation was carried out with little of the output losses suggested by a traditional Phillips curve, consistent with Sargent's study of interwar hyperinflations. Finally, Bental and Eckstein use their model to quantify the welfare gains associated with the disinflation.

My comments will focus on three aspects of the study: the fit of model, the Phillips curve trade-off, and the welfare calculation.

One of the paper's empirical findings is that the log-log money demand function:  $\log(m_t/c_t) = \alpha + \beta \times \log[(1 + R_t)/R_t]$  fits the Israeli data better than the semi-logarithmic specification  $\log(m_t/c_t) = \alpha + \beta \times R_t$ . The paper's Figure 7 shows this result quite clearly. The standard error for the log-log specification is 15 percent, while the standard error for the semilog specification is 34 percent. This improvement in fit is much more difficult to see in economics with more moderate variations in inflation. For example, comparing the same two money demand specifications for U.S. annual M1 data over the twentieth century yields standard errors of 15 percent

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for the log-log specification and 14 percent for the semilog specification. Thus, for the United States both models apparently fit the data equally well.

While Figure 7 shows that the basic model fits the data well, there are two sets of points that fit less well. The first are the poststabilization observations highlighted in the graph and discussed at length in the paper. The second set are the points directly above the poststabilization cluster. This second set of errors correspond to the rapid increase in nominal interest rates that occurred in 1979–80 (see Figure 6). It would be interesting to know more about the apparent unusually high money demand during this period.

The authors carefully discuss potential explanations associated with the shortfall of money demand post-1985. One detailed explanation is the subject of their section 6. The basic idea is that during the high-inflation period, the possibility of stabilization and associated decline in future inflation leads to higher money demand. After stabilization occurs, there is no longer the possibility for additional future sharp decreases in inflation and this leads to a lower money demand. This is certainly plausible, but it does not explain the puzzle apparent in Figure 7, which shows money demand conditional on nominal interest rates. Since nominal interest rates reflect expected future inflation, they incorporate the effect of potential stabilization. Alternatively, the author's analysis in section 6 does not change the consumer's utility function (1) and budget constraint (2) and these are what generate the money demand function (12). Thus, the poststabilization shortfall in money demand remains a puzzle.

As the authors note, the stabilization of late 1985 was associated with little employment loss. Industrial production data suggest that there was little output loss as well. The rate of growth of industrial production was 3.4 percent in 1984, 1.3 percent in 1985, and 5.0 percent in 1986. The corresponding average quarterly inflation rates were 44 percent, 25 percent, and 4 percent. A closer look at 1985 shows a dramatic 23 percent decline in the third quarter, but a rebound of 20 percent in the fourth quarter. The output loss of the stabilization was apparently short lived.

I want to end with two remarks about the authors' inflation welfare cost calculation. In their model the welfare costs of inflation are reflected in the reduction of real balances held by consumers. These can be thought of as transactions or "shoe-leather" costs which in modern economics reflect resources associated with efficient cash management. Unfortunately, it is difficult to evaluate the authors' calculation because it is difficult to directly measure the influence of inflation on these resource costs. For example, in the United States, one way to measure the resource cost of inflation is to measure its effect on the share of resources flowing to the "finance" sector. The share of total employment in this sector trended up from 2.4 percent in the early 1970s to 3.2 percent in the mid-1980s before falling back to 2.8 percent in the mid-1990s. While there are variations in this employment trend, they are hard to associate with variations in inflation. For example, the employment trend continued steadily after the disinflation of the early 1980s and only turned around after the 1987 stock market crash.

Finally, even if the traditional money demand costs of inflation were found to be

low (which they aren't in this paper), there are other important costs of inflation. For example, Feldstein (1997) argues that the first-order costs of inflation are those associated with distortions in the tax system, and calculates enormous welfare gains from moving from 3 percent inflation to price stability.

In summary, Bental and Eckstein have provided convincing empirical evidence of a stable money demand relation during a period that incorporates the extremes of price inflation. All students of money demand and hyperinflation will benefit from reading it carefully.

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### *Comment on* ON THE FIT OF A NEOCLASSICAL MONETARY MODEL IN HIGH INFLATION: ISRAEL 1972–1990, *by Timothy S. Fuerst*

During hyperinflation the amount of real cash balances changes drastically. At first sight these changes may appear to reflect changes in individuals' preferences for real cash balances—that is, shifts in the demand function for the balances. But these changes in real cash balances may reflect instead changes in the variables that affect the desired level of the balances. . . . The hypothesis [is] that changes in real cash balances in hyperinflation result from variations in the expected rate of change in prices. (pp. 29–33)

The above quotation is drawn from Phillip Cagan's (1956) classic study, "The Monetary Dynamics of Hyperinflation," first published some four decades ago. Cagan's conclusion is truly remarkable—out of the apparent utter chaos typical of hyperinflations, Cagan argues that there is stability, a stable relationship between real money demand and expected inflation. The work of Benjamin Bental and Zvi Eckstein is in this Cagan tradition. The dates and places have changed, but the underlying result remains the same: the high inflation experienced by Israel in the recent past is a manifestation of a stable money demand curve interacting with a rapidly

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growing supply of money. In these remarks, I will first review the theoretical arguments for stability (arguments that are as applicable to Cagan as they are to Bental-Eckstein), and then turn to a few more pointed comments about Bental-Eckstein's contribution.

### 1. Stability?

In the volume *Studies in the Quantity Theory of Money*, the article immediately preceding Cagan's study of hyperinflations is Friedman's (1956) restatement of the quantity theory. Approaching the issue from a portfolio perspective, Friedman argues that the demand for money is a function of a long laundry list: total wealth, the ratio of nonhuman to human wealth, the nominal return on equity, the nominal return on physical goods, and the nominal return on short- and long-term bonds. Friedman suggests that this is a stable empirical relationship, but given the forecasting problem subsumed into, for example, the human wealth variables, there is little hope that Friedman will prove to be correct.

In sharp contrast, Bental-Eckstein's money demand function [given by their equation (13)] is a function solely of current consumption and the current short-term bond rate. The linkage between these two views of money demand is the permanent income hypothesis: money demand is a function of wealth and these other asset returns *only* through their effect on the consumption decision. This approach separates two logically distinct decisions: (i) the consumption-savings choice, and (ii) the cash balances versus number-of-trips-to-the-bank choice. This transactions approach to money demand yields a relationship that depends on only two currently observable variables, and thus provides some confidence that this relationship might be stable.<sup>1</sup> It is with this theoretical hunch for stability in hand that we can now turn to the work of Bental-Eckstein.

### 2. Details of Bental-Eckstein's Work

Bental-Eckstein present a partial equilibrium model in which infinitely lived households have preferences over consumption, leisure, and real money balances. The first-order conditions to the problem include a demand for money [equation (13)] and a supply curve for labor [equation (15)]. To begin, I have a few specific comments on the functional forms used to generate these relationships.

One unusual feature of the money demand function is that the interest elasticity is decreasing in the rate of interest. This is an artifact of the timing implied by the model. Real money balances at the end of the period enter into the utility functional. On intuitive grounds it seems that a more plausible choice would be to assume that beginning-of-period balances enter into the utility function. This minor change will transform equation (13) into a demand function with a constant interest elasticity of unity. However, this leads to a second problem. I know of no estimates of money demand elasticity that are this large. Lucas (1994), for example, reports an elasticity

1. McCallum and Goodfriend (1987) and Lucas (1988) make a similar point.

of .5.<sup>2</sup> In summary, I would prefer to see a utility function in which the constant elasticity of substitution between consumption and cash balances is much less than unity.<sup>3</sup>

A fundamental problem with the labor supply curve is that it is unaffected by the nominal rate of interest. This is simply implausible. My willingness to trade leisure for wage income depends on the ease with which I can turn wage income into consumption. The essential characteristic of a monetary economy is that it requires real money balances to make this transformation. Hence, my marginal rate of substitution must depend upon the level of real cash balances that I hold. This is quite clear if one posits an explicit shopping time formulation:  $U = U(c, 1 - L - s)$ , where  $L$  is working time, and  $s = \psi(c, m)$  is shopping time. In this case, it is clear that the marginal rate of substitution between consumption and leisure will depend upon the level of real balances (and in turn on the nominal rate of interest). Under standard assumptions, increases in the rate of interest will lower labor supply. A casual observation of Figures 6 and 8 suggest such a link.

To sum up these last two observations, I would prefer to see an analysis using a more general utility functional, one that is more consistent with our understanding of the transactions process. For example,  $U(c, m, 1 - L) = V[w(c, m), 1 - L]$ , where  $w$  is homothetic. The properties of  $w$  would determine money demand elasticity, while the marginal rate of substitution between consumption and leisure would depend upon real cash balances.

Analogous to their discussion of money demand and labor supply, it would be instructive for Bental-Eckstein to provide a discussion of the model's prediction for the nominal rate of interest. Equation (4) of the household's first-order conditions is essentially the Fisherian decomposition of the nominal interest rate into an intertemporal marginal rate of substitution and an inflation premium. Figures analogous to Figures 6 and 8 would be helpful here.

Finally, one intriguing fact from the paper is "the dramatic decrease of the reserve requirements" at the end of the inflationary period. Before turning to the expectational-based stories of section 6 to explain the apparent shift in money demand after the inflation, I would suggest that the authors pursue an analysis of the effect of the change in reserve requirements. A "dramatic decrease of the reserve requirements" would surely have an effect on the demand for base money.

### 3. Conclusion

The essential theme of Cagan and Bental-Eckstein's work is documentation of stability in an apparently unstable environment. This conclusion exhibits *once more* the power of economic science, and at the same time points a clear finger of blame.

2. Chang (1996) calls into question many of these estimates. In an interesting theoretical study, he demonstrates that the homogeneity properties of money demand imply that the interest elasticity must exceed the transactions elasticity.

3. As an aside, Cagan's exponential money demand function as well as the log-log function are equally consistent with Sidrauski's suggestion to add money to the utility function. In other words, there exists a utility function that generates Cagan's money demand function. Also, Cagan (1956) suggests that a log-log specification may better fit the data: "To be consistent with the data, this hypothesis requires . . . that the value of [the semi-elasticity] shall decline as the expected change in prices increases" (p. 55).

These inflations are not an accident. I can think of no better way to end these remarks than with the way I began them, by quoting Cagan:

[Previous] theories [of hyperinflation] postulate that a rise in prices results from an increase in wages or prices of imported goods and precedes increases in the quantity of money. This study points to the opposite sequence and indicates that an extreme rise in prices depends almost entirely on changes in the quantity of money . . . and involves the motives of governments, with whom the authority to open and close the spigot of note issues ultimately lies. (p. 91)

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