Appendices: What Explains Growing Gender and Racial Education Gaps?

by

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Appendix A: Details of the Life-Cycle Model

Our model is based on Eckstein, Keane and Lifshitz (2019). The main text describes how we extend that model to: (1) incorporate a broader definition of parental background, to include not just parent education but also marital status and immigration status, (2) to allow labor and marriage market opportunities to vary by gender, cohort and race/ethnicity, (3) to allow tastes for marriage to depend on parents' marital status, and (4) to include welfare participation as a choice and allow welfare rules to vary over time. Here we describe the overall structure of the model, that is very close to that in EKL.

A1. Overview

Agents enter the model at age 17 as single individuals in school. Both men and women make annual private decisions about school continuation and work. In addition, women make annual decisions about fertility, and single mothers decide whether to participate in a welfare program (if eligible). We assume only single people can attend school.^{[1](#page-1-0)} Retirement is enforced at age *T*=65, at which point agents receive a terminal value function. The men and women in the model also interact in a marriage market, so they can choose to form (and later dissolve) couples. Once a couple is formed, decisions about labor supply and fertility are made jointly.

To make marriage decisions, individuals compare the values of the married and single states. We first describe the problem of single individuals, followed by the problem of married couples. We are then in a position to explain how we model the marriage market.

A2. The Decisions of Single Households

First we describe the optimization problems of single (i.e., unmarried) women and men. Let *t* denote the annual time period, and let $j = f$, *m* denote gender.

A.2.1. Income and Consumption of Singles

The gross income of a single man is simply $GY_t^m = w_t^m h_t^m$, where w_t^m denotes his annual wage rate and $h_t^m \in \{1, 0.5, 0\}$ indicates whether he works full-time, part-time or not at all. The gross income of a single woman is:

$$
(A1) \quad GY_t^f = w_t^f h_t^f + wb(N_t, w_t^f h_t^f, G_t) \cdot g_t + CS_t \cdot a_t \cdot I[N_t > 0]
$$

The second term in (A2) captures welfare benefits. g_t is a 1/0 indicator for the decision to go on welfare. $wb(N_t, w_t^j h_t^j, G_t)$ denotes the benefit level to which a single mother with N_t children and income $w_t^f h_t^f$ is eligible. The state variable G_t is number of years the woman has received benefits in the post-1996 period. We include this to capture time limits on eligibility.

¹ School attendance by married people is rare. We rule out school attendance after age 30 for the same reason.

The third term in (A1) is child support *CS* a single woman with children may receive. a_t is a 1/0 indicator for receiving child support, which occurs with probability $P(a_t = 1)$, which is a free parameter that we estimate.^{[2](#page-2-0)} We let $CS_t = \exp(\varepsilon_t^a)$ where $\varepsilon_t^a \sim N(\mu_a, \sigma_\varepsilon^a)$.

The net income of a single person is given by:

$$
(A2) \tY_t^j = GY_t^j - \tau_t^S(w_t^j h_t^j, N_t) \t j = f, m
$$

where $\tau_t^S(w_t^j h_t^j, N_t)$ is the time *t* tax function for single individuals calculated using the tax rules described in the Appendix C. Thus, the budget constraint for a single person is simply:

$$
(A3) \quad C_t^j = (1 - \kappa(N_t))Y_t^j
$$

where $\kappa(N_t)$ is the cost of children which we specify as a fraction of income.^{[3](#page-2-1)} Note that both single men and women may have children $(N_t > 0)$. These may be children from a previous marriage or, in the case of single women, children born outside of marriage.

A2.2. Utility of a Single Woman

The per-period utility function of a single female is given by:

(A4)
$$
U_t^f(\Omega_{ft}) = \left(\frac{1}{\alpha}(C_t)^{\alpha} + L_j(l_t) - \Psi g_t + \pi_t p_t + A_f^S Q(l_t, 0, Y_t, N_t)\right)(1 - s_t) + U_{ft}^S s_t
$$

If a woman is a worker who has left school her utility depends on consumption, leisure, welfare participation, pregnancy and children via the term $\frac{1}{\alpha}(C_t)^{\alpha} + L_j(l_t) - \Psi g_t + \pi_t p_t + A_f^s Q(\cdot)$. In contrast, if she is in school ($s_t = 1$) she simply gets the utility from attending school U_{ft}^S that we defined in the main text in equation (1).

The asymmetry in how we treat students vs. workers is motivated by the fact that we cannot measure leisure time for students in a way comparable to that for workers, [4](#page-2-2) as well as a desire to avoid modelling how consumption is financed by students.^{[5](#page-2-3)} Hence, consistent with prior work like Keane and Wolpin (1997), we simply define a "utility while in school" variable.

The first term $\frac{1}{\alpha}(C_t)^{\alpha}$ in utility for workers is a CRRA in consumption with curvature parameter α . The second term, $L_i(l_t)$, captures the value of leisure and home production. The third term Ψ is a disutility of welfare participation. The fourth term captures the utility (or disutility) from a pregnancy $(p_t = 1)$, and the fifth term captures utility from the quality and quantity of children. We now discuss the $2nd$ through $5th$ terms in more detail:

² The child support parameters are identified by the fraction of unemployed single mothers who go on welfare.

³ The equivalence scale implies that $\kappa(N) = 0.194$, 0.293, 0.367 and 0.423 if $N = 1, 2, 3$ or 4, respectively.

⁴ While we can see hours of market work, we cannot measure hours spent on school work.

⁵ Consumption while in school may be financed by a combination of parental transfers, financial aid, part-time work, etc. (see Keane and Wolpin (2001). It is beyond the scope of our paper to model all these possibilities.

A.2.2.1. *Tastes for Leisure and Value of Home Production*

We write the utility from leisure $L_i(l_t)$ as:

$$
(A5) \quad L_{jt}\left(l_t^j\right) = \frac{\Gamma_{jt}}{\gamma} \left(l_t^j\right)^{\gamma} + \xi_{jt} l_t^j \qquad \gamma < 1, \alpha < 1
$$

The parameter Γ_{it} which must be positive, shifts tastes for leisure. We allow Γ_{it} to depend on education and on health status. For women it also depends on pregnancy p_t , as in:

(A6)
$$
\Gamma_{mt} = \vartheta_{0m} + \vartheta_{1m} E_t + \vartheta_{2m} H_t
$$
 and $\Gamma_{ft} = \vartheta_{0f} + \vartheta_{1f} E_t + \vartheta_{2f} H_t + \vartheta_{3f} p_t$

The second term in (A5) captures stochastic variation in the marginal utility of leisure. This is denoted by $\xi_{jt} l_t^j$ where ξ_{jt} is a random variable. We assume shocks to tastes for leisure (i.e., home time) follow a stationary $AR(1)$ process, as in:

(A7)
$$
\ln(\xi_{jt}) = \tau_{0j} + \tau_{1j}\ln(\xi_{j,t-1}) + \tau_{2j}p_{t-1} + \varepsilon_{jt}^l
$$
 where $\varepsilon_{jt}^l \sim \text{iidN}(0, \sigma_{\varepsilon}^l)$

where $0 < \tau_{1j} < 1$. Arrival of a new child at time *t* (i.e., $p_{t-1} = 1$) shifts tastes for home time by τ_{2i} . We expect the marginal utility of home time will jump up when a newborn arrives, particularly for women (i.e., $\tau_{2f} \gg 0$), capturing the desire to spend time with the child and an increase in time required for home production. Afterward, provided no new children arrive, tastes for home time gradually revert to normal, as τ_{1f} < 1. This lets us generate the decline in women's employment after childbirth, as well as their subsequent gradual return to the labor force.^{[6](#page-3-0)} The stochastic terms ε_{jt}^l generate heterogeneity in these response patterns.

A.2.2.2. *Disutility of Welfare Participation*

The parameter Ψ in (A4) is a disutility of welfare participation. It may capture social "stigma," as well as time and effort costs arising from the various work/training/search and reporting requirements imposed on welfare participants. The welfare reform of 1996 can be thought of as making these requirements more stringent, so we let Ψ increase after 1996. Recall that a time limit of welfare eligibility also goes into effect in 1996. See section III.F of the text.

A.2.2.3. *Utility from Pregnancy*

The utility from pregnancy π_t is given by:

(A8)

\n
$$
\pi_t = \pi_{00} I(N_t = 0) + \pi_{01} I(N_t = 1) + \pi_{02} I(N_t \ge 2) + \pi_1 (1 - M_t) + \pi_2 A G E_{ft}
$$
\n
$$
+ \pi_3 p_{t-1} + \varepsilon_t^{p0} \cdot I(N_t = 0) + \varepsilon_t^{p1} \cdot I(N_t > 0)
$$

where $\varepsilon_t^{pk} \sim \text{iidN}(0, \sigma_{\varepsilon}^{pk})$ for $k=0,1$. Here π_t is a function of the number of children already present, marital status (M_t) is a 1/0 indicator for marriage), and lagged pregnancy. The variance of the error is allowed to depend on whether the woman already has children.

⁶ The intercepts ϑ_{0i} in (A6) and τ_{0i} in (A7) are not separately identified, so we set $\vartheta_{0i} = 0$ for $j=m.f$.

A.2.2.4. *Utility from Quantity and Quality of Children*

Finally, consider the function $Q(\cdot)$ that determines the utility a person receives from children. This depends on the quantity of children, and also on inputs that increase child quality: the home time (leisure) of both parents and the income of the parents – see Becker and Lewis (1973). We assume $Q(\cdot)$ is a CES function of the inputs, as follows:

(A9)
$$
Q(l_t^f, l_t^m, Y_t^M, N_t) = (a_f(l_t^f)^\rho + a_m(l_t^m)^\rho + (1 - a_f - a_m)(\kappa(N_t)Y_t^M/N_t)^\rho)^{1/\rho} \cdot N_t^{\rho 0}
$$

Here $\kappa(N_t) Y_t^M/N_t$ is spending *per child*, which is not a choice but rather determined by a square root equivalence scale. The parameter A_j^S in the utility function (4a) is a scale parameter that multiplies $Q(\cdot)$. This parameter is allowed to differ in the married state (see below). For single women with children we have $Q(l_t, 0, Y_t, N_t)$, so the male time input is set to zero.

A2.3 Utility of a Single Man

The utility function for single men is analogous to (A4), except they do not have the welfare participation (g_t) and pregnancy (p_t) options, so the $-\Psi g_t + \pi_t p_t$ term drops out. And utility from children is $Q(0, l_t, Y_t, N_t)$.

A2.4 Choice Specific Value Function for Single Men and Women

We can now write the choice-specific value function for single females. We let Ω_{ft} denote her current state. We assume for now the women chooses to stay single, and conditional on staying single she chooses school, labor supply, pregnancy and welfare participation:

(A10)
$$
V_t^f(s_t, l_t, p_t, g_t | \Omega_{ft}) = \left(\frac{1}{\alpha} (C_t)^{\alpha} + L_f(l_t) - \Psi g_t + \pi_t p_t + A_f^s Q(l_t, 0, Y_t, N_t)\right) (1 - s_t) + U_{ft}^s s_t + \delta E_{MAX} V(\Omega_{f, t+1})
$$

Here δ is the discount factor and $E_{MAX}V(\Omega_{f,t+1})$ is the expected maximum of the *t*+1 value function, given the next period state $\Omega_{f,t+1}$ that is determined by the current state Ω_{ft} and the current choice $\{l_t, p_t, s_t, g_t\}$, as well as random factors.

One of these random factors is whether the woman receives a marriage offer at *t*+1, and whether that offer is good enough for her to decide to get married. The Emax function takes into account that the person may get married at *t*+1. It takes the form:

(A11)
$$
E_{MAX}V(\Omega_{f,t+1}) = E_{MAX}(M_{t+1}V_{t+1}^{fM}(\Omega_{m,t+1}, \Omega_{f,t+1}) + (1 - M_{t+1})V_t^f(\Omega_{f,t+1}))
$$

Notice that if $M_{t+1} = 0$ the future value function is simply $V_t^f(\Omega_{f,t+1})$. But if $M_{t+1} = 1$ then the future value function is V_{t+1}^{fM} where the superscript fM denotes the value function of a married women. We will define the value functions for married men and women below.

The choice-specific value functions $V_t^m(l_t, s_t | \Omega_{mt})$ for single men are analogous, except they do not have the welfare participation (g_t) and pregnancy (p_t) options, so the $-\Psi g_t + \pi_t p_t$ term drops out. And utility from children is $Q(0, l_t, Y_t, N_t)$.

A.2.5. The Maximized Value Functions for Single Men and Women

Now we consider the optimization problem of singles. In Section III.E we discuss the marriage market, but we must first consider decision making *conditional* on being single – i.e., the state where no marriage offer is available or where it has already been declined.

Let $V_t^m(\Omega_{mt})$ and $V_t^f(\Omega_{ft})$ denote the maximized value functions of single males and females in period *t*. Let S_t^m and S_t^f denote the feasible set of choice options for a single male and female in period *t*, respectively. As we see in Section III.C.3 of the text, workers receive job offers probabilistically, so S_t^m and S_t^f may not include all possible levels of work hours and leisure. To proceed, for women and men we have, respectively:

(A12)
$$
V_t^f(\Omega_{ft}) = \max_{\{l_t, p_t, s_t, g_t\} \in S_t^f} V_t^f(l_t, p_t, s_t, g_t | \Omega_{ft})
$$

(A13)
$$
V_t^m(\Omega_{mt}) = \max_{\{l_t, s_t\} \in \mathcal{S}_t^m} V_t^f(l_t, s_t | \Omega_{mt})
$$

These value functions appear below in equations (A21) and (A29) that govern divorce and marriage decisions, respectively.

A.3. The Decisions of a Married Couple

In our model, utility functions exist at the *individual* level, and are not fundamentally altered by marriage. Consistent with this, we specify the utility functions of married agents to be as similar as possible to those of single agents. We assume a collective model of household decision making, as in Mazzocco (2007), Chiappori (1992), Apps and Rees (1988). Thus, within marriage, collective household decisions are made by constrained maximization of a weighted average of the individual partners' utility functions.

Conditional on marriage, couples have three choice variables: Leisure of the husband and wife, $\{l_t^m, l_t^f\}$, and pregnancy, $p_t \in \{0, 1\}$. Pregnancy leads deterministically to arrival of a child at *t*+1. Couples also make annual decisions about divorce/marriage continuation. We ignore this for now and focus on the joint decisions of couples' *conditional* on marriage.

A3.1. *Budget Constraint of a Married Couple*

Married couples have total gross income $G Y_t^M$ given by:

(A14) = (ℎ + ℎ)

Here w_t^j and h_t^j for $j = f$, *m* are annual full-time wage rates. We will use the *M* superscript

throughout to indicate values for married individuals.^{[7](#page-6-0)} Net income is Y_t^M given by the equation:

(A15)
$$
Y_t^M = G Y_t^M - \tau_t^M ((w_t^m h_t^m + w_t^f h_t^f), N_t),
$$

where $\tau_t^M(\cdot, \cdot)$ is the tax function for married couples based on the time *t* tax rules. We model the US federal tax system in detail, including deductions, exemptions, EITC, and the joint taxation of couples (see Appendix C). We assume perfect foresight regarding tax rules.

The household budget constraint takes the form:

$$
(A16) \tC_t^M = (1 - \kappa(N_t))Y_t^M
$$

Here $\kappa(N_t)$ is the fraction of Y_t^M spent on children, based on a square root equivalence scale.^{[8](#page-6-1)} We assume a static budget constraint as it is computationally infeasible to add saving in addition to our other state variables. However, the terminal value function (at age 65) proxies for how labor supply affects Social Security and retirement assets, so these key aspects of savings do enter our model in a reduced form way.

A3.2. *Utility Function of Married Individuals*

The period utility of a married person of age *t* and gender *j* in state Ω_{it} is given by:^{[9](#page-6-2)}

(A17)
$$
U_t^{jM}(\Omega_{jt}) = \frac{1}{\alpha}(\psi C_t^M)^{\alpha} + L_{jt}(l_t^j) + \theta_t^M + \pi_t^M p_t + A_j^M Q(l_t^f, l_t^m, Y_t^M, N_t) \quad j = m, f
$$

We assume household consumption C_t^M is a "public" good. The full amount C_t^M enters the utility of both the husband and wife. The parameter $\psi \in (\frac{1}{2}, 1)$ captures household economies of scale in consumption. The square root equivalence scale gives $\psi = 1/\sqrt{2} = 0.707$, so a couple needs 41% more expenditure than a single person to obtain an equivalent consumption level.

Notice that most terms in (A17) are also present in the utility functions for singles in (A4). The exception is the third term (θ_t^M) that captures the utility from marriage itself. If a single woman is not in school $(s_t = 0)$, her utility function is fundamentally identical to that of a married woman, as one can see by comparing (A4) and (A17). The only differences are that in (A4) consumption is individual specific (i.e., $\psi = 1$), utility from marriage is (of course) dropped, utility from children is allowed to differ from the married state $(A_j^s \neq A_j^M)$, the hometime of the husband is set to zero in the *Q* function, and welfare participation is an option.

⁷ We could include an unemployment benefit (or value of home production) in (A14) but it would not be separately identified from the value of leisure parameters Γ_{it} in (A6).

⁸ For a household with two adults, the square root scale implies that $\kappa(N) = 1 - \sqrt{2/(2 + N)}$.

⁹ The state vector Ω_{jt} contains four variables that are relevant for $U_t^M(\Omega_{jt})$. These are N_t and μ_{jt} , as well as parent's education and marital status, and health. The state vector Ω_{it} contains several additional variables, whose role will only become clear after the full model is laid out. Thus, we defer giving the complete list of elements of Ω_{it} until we finish expositing the full model and turn to discussing the DP problem solution (Section IV).

Note that the utility from pregnancy, π_t^M , defined in (A8), contains nothing individual specific. That is, it does not differ between the two partners in a couple. We assume pregnancy decisions are made jointly by the couple, and each party gets the same utility from the decision.[10](#page-7-0) Next we describe utility from marriage in more detail:

A3.2.1. *Match Quality and the Utility of Marriage*

The utility from marriage (θ_t^M) or match quality is given by:

$$
(A18) \ \theta_{tj}^M = d_1 + d_2 \cdot I[E^m - E^f > 0] + d_3 \cdot I[E^f - E^m > 0] + d_4(H_t^m - H_t^f)^2 + \varepsilon_t^M
$$

where $\varepsilon_t^M \sim \text{iidN}(0, \sigma_\varepsilon^M)$ and E^j denotes education, rank ordered as high school dropout (HSD), high school (HSG), some college (SC), college (CG) and post-college (PC), and $H_t^j \in \{1, 2\}$ denotes health (i.e., good or poor). The $2nd$ and $3rd$ terms capture assortative mating on education. $I[E^m - E^f > 0]$ indicates the man has greater education than the woman, and $I[E^f - E^m > 0]$ indicates the reverse. If $d_3 < 0$ people are averse to matches where the woman has more education. The 4th term captures assortative mating on health. If d_4 <0 people prefer matches where partners have similar health. Finally, ε_t^M is a transitory shock to match quality.

A.3.3. Choice Specific Value Functions of Married Individuals

We are now able to write the choice-specific value functions for married *individuals*. These depend on both a person's own state and that of their partner:

$$
(A19) V_t^{jM}(l_t^m, l_t^f, p_t | \Omega_{mt}, \Omega_{ft}) = \frac{1}{\alpha} (\psi C_t^M)^{\alpha} + L(l_t^j) + \theta_t^M + \pi_t p_t + A_j^M Q(l_t^f, l_t^f, Y_t^M, N_t)
$$

$$
+ \delta E_{MAX} (M_{t+1} V_{t+1}^{jM} (\Omega_{m,t+1}, \Omega_{f,t+1}) + (1 - M_{t+1}) V_t^j (\Omega_{j,t+1})) \qquad j = f, m
$$

The current payoff simply repeats (A17). The future component in (A19) consists of two parts, corresponding to whether the marriage continues at $t+1$ or not. The term $V_{t+1}^{jM}(\Omega_{m,t+1}, \Omega_{f,t+1})$ is the value of next period's state for partner *j* if the marriage continues. The term $V_t^{\prime}(\Omega_{jt+1})$ is the value of next period's state for partner *j* if he/she becomes single (i.e., a divorce occurs). These were defined in equations (A12) and (A13). We discuss the divorce decision below.

The *t*+1 state depends on the current state $\{\Omega_{mt}, \Omega_{ft}\}\$ and current choices $\{l_t^m, l_t^f, p_t\}\$ via the laws of motion of the state variables. δ is the discount rate and $E_{MAX}(\cdot)$ is the expectation taken over elements of the *t*+1 state that are unknown at *t*. These include M_{t+1} , $\{\varepsilon_{jt+1}^l\}$ for $j = m$, *f*, ε_{t+1}^M and ε_t^p , as well as realizations of wage shocks and job offers. We defer a detailed discussion of these until Section III.C which describes the labor market.

¹⁰ Of course, one could imagine individuals in a couple getting different utilities from a pregnancy decision, but we cannot infer such differences from the data so we ignore them.

A3.4. *Household Decision Making for Married Couples*

In our collective model the household value function is given by:

(A20)
$$
V_t^M(l_t^m, l_t^f, p_t | \Omega_{mt}, \Omega_{ft}) = \lambda V_t^{fM}(l_t^m, l_t^f, p_t | \Omega_{mt}, \Omega_{ft}) + (1 - \lambda)V_t^{mM}(l_t^m, l_t^f, p_t | \Omega_{mt}, \Omega_{ft})
$$

Here λ and (1- λ) are Pareto weights. We set $\lambda = 0.5$ for simplicity.¹¹ The V_t^{jM} for $j=f$, *m* are the choice-specific value functions of the *individual* married partners. The Ω_{jt} for $j=f$, *m* are the

state vectors of these individuals. Couples seek a choice vector $\{l_t^m, l_t^f, p_t\}$ to maximize (A20), subject to the constraint that both parties prefer marriage over the outside option of divorce.^{[12](#page-8-1)}

Recall that $V_t^m(\Omega_{mt})$ and $V_t^f(\Omega_{ft})$ denote the maximized value functions of single males and females in period *t*, see equations (A12)-(A13). Utility is not transferable, so a divorce occurs if the value of the outside (single) option exceeds the value of marriage for *either* party. Let *F* denote the feasible set of choice options. A choice vector $\{l_t^m, l_t^f, p_t\} \in \mathcal{F}$ if:

(A21)
$$
V_t^{jM}(l_t^m, l_t^f, p_t | \Omega_{mt}, \Omega_{ft}) \geq V_t^j(\Omega_{jt}) - \Delta_{jt} \text{ for } j = f, m
$$

where Δ_{jt} is the cost of divorce. If no choice vector $\{l_t^m, l_t^f, p_t\}$ satisfies (11) then $\mathcal{F} = \emptyset$.

The cost of divorce depends on the number of children, $\Delta_{jt} = \alpha_4^j + \alpha_5^j N_t$. This cost is fixed across cohorts. Many US States switched to unilateral divorce laws in the 1970s, which lowered divorce costs. Both Voena (2015) and Bronson (2015) find this had important impacts on behavior. But our oldest cohort (1960) entered the marriage market on the 1980s.

We can now formally define the solution to the maximization problem. Denote the vector of household choices that maximize equation (A20) as $\{l_t^{m*}, l_t^{j*}, p_t^*\}$. That is,

$$
\{l_t^{m*}, l_t^{f*}, p_t^*\} = \begin{cases} arg \max_{\{l_t^m, l_t^f, p_t\} \in \mathcal{F}} V_t^M(l_t^m, l_t^f, p_t | \Omega_{mt}, \Omega_{ft}) & \text{if } \mathcal{F} \neq \emptyset \\ \emptyset & \text{if } \mathcal{F} = \emptyset \end{cases}
$$

The form of (A20) insures that $\{l_t^{m*}, l_t^{j*}, p_t^{*}\}$ is a Pareto efficient allocation. If one or more parties prefer to remain single for all possible $\{l_t^m, l_t^f, p_t\}$ then $\mathcal{F} = \emptyset$ and a divorce occurs.

The maximized value function of a married *individual* in state Ω_{it} is given by:

(A22)
$$
V_t^{jM}(\Omega_{mt}, \Omega_{ft}) \equiv \begin{cases} V_t^{jM}(l_t^{m*}, l_t^{f*}, p_t^* | \Omega_{mt}, \Omega_{ft}) & \text{for} \quad j = f, m \quad if \quad \mathcal{F} \neq \emptyset \\ -\infty & \text{for} \quad j = f, m \quad if \quad \mathcal{F} = \emptyset \end{cases}
$$

The maximized value function depends on both the own state Ω_{it} and that of the partner. Note

¹¹ This simple specification is similar to Voena (2015), who considers a household planning problem with a unilateral divorce regime. We discuss our decision to use this simple specification in EKL (2019) Section V.

¹² If we take the *unconstrained* maximum of (A20), we might obtain a solution for $\{l_t^m, l_t^f, p_t\}$ where only one party prefers to stay married. In a transferable utility framework, a marriage may persist in such a case, using transfers between partners. We do not adopt this approach, as such transfers would be difficult to enforce.

that if $\mathcal{F} = \emptyset$ then no action exists such that person *j* can be married at time *t*, so a divorce occurs. Then $V_t^{jM} = -\infty$, so behavior is governed solely by the single value function $V_t^j(\Omega_{jt})$.

We discuss the marriage market and decisions to get married in Section A.5. First, we need to describe health, so we have all the state variables relevant for marriage offers.

A.4. Health Status

There are substantial disparities across race/ethnic groups in health and mortality, so it is important to account for these. We assume health evolves over the life-cycle according to a two-state Markov chain, where $H_{it} \in \{1,2\}$ indicate good and fair/poor, respectively. The transition probabilities differ by cohort and race/ethnicity. We assume health is an *exogenous* process, so it can be estimated outside the model.

Health plays several important roles in our model. For example, we require people to retire by age 65, but declining health may induce them to retire earlier, as health affects both tastes for work (A6) and job offer probabilities (4). Health is also a dimension on which people sort in the marriage market (A18), and it shifts tastes for pregnancy (A8). Furthermore, as we assume health is not affected by employment, marriage or fertility decisions, it generates exogenous variation in these decisions (*given* our model).

A.5. The Marriage Market

 The final component of the model is the marriage market. Single people may receive marriage offers, and they choose to become married if they draw a good enough match. To make this decision, they must compare the value of remaining single to the value of entering the married state. This section describes how the matching process works.

A5.1. *Marriage Offers*

 At the start of a period a single individual may receive a marriage offer. Denote the probability of receiving an offer as $p_j^H(\Omega_{jt})$ for $j = f, m$. We assume the probability is given by a binomial logit model that depends on age and age-squared, whether a person is below 18, and whether a person is in school. The age effects differ by gender, race and cohort.

 A marriage offer is characterized by a vector of attributes of a potential spouse, denoted by ℳ*jt*. We assume marriage offers always come from a potential spouse of the same age (*t*). This is necessitated by technical issues that arise in solving the dynamic programming problem (see Appendix D1 for details). We do not think this assumption will have too great an effect on the results, because the large majority of married couples are in fact close in age.^{[13](#page-9-0)} It is

¹³ For the cohorts of 1960-1980, the age gap between partners is below 5 years for 79% of all couples. It is below 7 years for 88% of couples, and below 10 years for 94% of couples.

convenient to describe the construction of marriage offers in three steps:

First, we draw the education of the potential spouse. We assume potential spouses have three possible education levels: high-school and below (HS, *ed = 0*), some college (SC, *ed = 1*) or college or above (C, *ed* = 2). The probability of receiving an offer from a potential spouse of the HS, SC or C type depends on a person's own education.^{[14](#page-10-0)}

 Specifically, if the individual gets a marriage offer, we draw the potential partner's education using a multinomial logit (MNL) with the following latent indices:

$$
v_{jt}^C = \eta_{0j}^C + \eta_{1j}^C \cdot I[ed^m - ed^f = 2] + \eta_{2j}^C \cdot I[ed^m - ed^f = 1] + \epsilon_{jt}^C
$$

(A23)

$$
v_{jt}^{SC} = \eta_{0j}^{SC} + \eta_{1j}^{SC} \cdot I[ed^m - ed^f = 1] + \epsilon_{jt}^{SC}
$$

High school is the base case with $v_{jt}^{HS} = 0$. The parameters η govern the probability that a person (of given education) receives offers from potential partners with different education levels. The *η* reflect both the supply of potential partners and tastes for partners of different types.^{[15](#page-10-1)}

Rather than solve explicitly for marriage market equilibrium, we estimate parameters *η* that, when combined with the rest of our model, generate (to a good approximation) the observed distribution of match outcomes between types of partners.[16](#page-10-2) Our method of moments estimation algorithm ensures that the assortative mating patterns predicted by the model are very close to those observed in the data – see Section IV.

Crucially, we let the *η* parameters differ by race/ethnicity and cohort. This captures different supplies of potential partners within each race/ethnic group and over time, as well as different and changing tastes for partners of different education levels.

Our approach allows us to side-step making explicit assumptions about intermarriage. We do this by searching for parameters *η* that enable us to match the frequencies with which both men and women in each race/ethnic group marry partners with each level of education, *irrespective* of the race/ethnic identify of the partners. For example, our estimation does not constrain the number of Hispanic women who marry college men to equal the number of

¹⁴ To simplify the MNL we combine the HSD and HSG levels into "HS," and the CG and PC levels into "C." Then, if a person draws "HS" we assign education level HSD or HSG to the potential partner according to the actual fraction in the data (by cohort and age). We do the same to convert "C" draws to CG and PC offers. ¹⁵ For example, suppose men have a strong preference for partners with similar education. Then a HSD woman may have little chance of receiving an offer form a college educated man, regardless of the supply of college educated men.

¹⁶ Our estimated *η* are therefore reduced form parameters that implicitly combine (i) structural parameters of preferences for different types of partners with (ii) endogenously determined supplies of partners. This approach has two key advantages: (1) It greatly simplifies estimation of the model relative to a case where we solve explicitly for the marriage market equilibrium, and (2) it allows us to avoid making detailed assumptions about how the marriage market works. The downside of course, is that we must assume the *η* are invariant to any policy experiments we may choose to consider.

married college-educated Hispanic men (or vice versa).^{[17](#page-11-0)}

Of course, substantial segregation of marriage markets along race/ethnic lines does exist, so we expect the *η* parameters to imply that black and Hispanic women have a much lower rate of receiving offers from college-educated men than white women (and to find similar differences for men). We find that such differences in marriage market prospects are important for explaining differences in behavior between race/ethnic groups.

Once the education is drawn, we calculate the potential work experience as the age of the individual minus his years of schooling minus 6. Then, we draw the remaining elements of \mathcal{M}_{it} . The six *observed* elements are drawn from the population distribution of all potential partners within a person's own age cell. These six elements of \mathcal{M}_{it} are partner's health, number of children, *PE*, *PM*, *PI* and lagged work. Their distributions are not conditional on unobservables, so we can obtain them from the raw data.

Note, for example, that we draw the 6 characteristics for potential spouses of black women from the actual distribution of black women's husbands, regardless of the race of the husband. So the distribution of characteristics accounts for inter-marriage.

Finally, the four *unobserved* elements of \mathcal{M}_{jt} are drawn from their population distributions as specified in the model. These are the potential partner's tastes for leisure ξ_{it} , labor market ability μ_j^W , transitory wage shock $\tilde{\varepsilon}_{jt}^W$, and the taste for marriage, ε_t^M . The stochastic terms ξ_{jt}, μ_j^W , $\tilde{\varepsilon}_{jt}^W$ and ε_t^M , are observed by both parties as part of the marriage offer. Both parties also understand which terms are permanent and which terms are only transitory.

Putting this all together, the marriage offer for a single female consists of the vector:

(A24)
$$
\mathcal{M}_{ft} = (E^m, X^m, H^m, N^m, PE^m, PM^m, h_{t-1}^m, \xi_{mt}, \mu_m^W, \tilde{\varepsilon}_{mt}^W, \varepsilon_t^M)
$$

Marriage offers for males (ℳ*mt*) have an analogous form.

A.5.2. *Marriage Decisions*

Given a marriage offer \mathcal{M}_{jt} , a single person can construct the vector $(\Omega_{ft}, \Omega_{mt})$ that characterizes the state of the couple if they marry. That is, $(\Omega_{jt}, \mathcal{M}_{jt}) \rightarrow (\Omega_{ft}, \Omega_{mt})$ for $j = f, m$. The potential partner also knows $(\Omega_{ft}, \Omega_{mt})$. Both parties calculate the value of marriage, denoted by $V_t^{jM}(\Omega_{mt}, \Omega_{ft})$ for $j = f, m$ in equation (A22). A marriage is formed if and only if: (A25) $V_t^{fM}(\Omega_{mt}, \Omega_{ft}) - \Delta(PM_f) > V_t^f(\Omega_{ft})$ and $V_t^{mM}(\Omega_{mt}, \Omega_{ft}) - \Delta(PM_m) > V_t^m(\Omega_{mt})$

¹⁷ The rate of college education among Hispanic men is very low, so if marriage markets were segmented by race/ethnicity we would expect Hispanic women to receive a low rate of offers form college men. But our estimated *η* parameters allow Hispanic women to receive offers from college-educated men at a higher rate – allowing implicitly for the possibility that they also receive offers from college educated white or black men.

Here $\Delta(PM_i)$ is a fixed cost of marriage that we allow to depend on the marital status of the parents of each partner, as in:

$$
(A26)\ \Delta(PM_j) = \alpha_{m0}^j + \alpha_{m1}^j PM_j \quad \text{for} \quad j = f, m
$$

This allows for the possibility that there is intergenerational transmission in tastes for marriage. If the pair decides to marry they proceed to make collective decisions about work and fertility as described in Section III.B. If the pair decides to remain single they individually make decisions about work, school and (for women) fertility as described in Section III.A.

A.6. Terminal Period and Retirement

The terminal period in the model is fixed at age 65, at which point everyone must retire. Of course, people can choose to stop working earlier if desired. By setting the terminal period at 65 we avoid the complications of modelling Social Security and the accumulation of retirement savings.[18](#page-12-0) To reduce computational burden, we assume the terminal value function $V_{T+1}^J(\Omega_{JT})$ at *T*=65 is a simple function of state variables – see Appendix F. Thus, the terminal value function accounts for retirement savings in a reduced form way.

¹⁸ For our four cohorts, the "normal" age for claiming Social Security (SS) benefits was gradually increased from 65 to 67. But workers can also opt to receive "early" SS benefits at age 62, subject to a penalty in the form of a reduced benefit level. To avoid having to model this decision, we chose to fit our model only to data up through age 61. By setting the terminal value function at age 65, we implicitly assume away the early SS option.

Appendix B: Detailed Description of the Data

B.1. The CPS data

We use the Annual Demographic Surveys (March CPS supplement) conducted by the Bureau of Labor Statistics and the Bureau of the Census. This survey is the primary source for detailed information on income and employment in the United States. A detailed description of the survey can be found at: [www.bls.census.gov/cps/ads/adsmain.htm.](http://www.bls.census.gov/cps/ads/adsmain.htm) Our data, for the years 1962-2023, was extracted using the [IPUMS.](https://www.ipums.org/) We restrict the sample to civilian adults, aged 17-60, and exclude those who are members of the armed forces or institutionalized.

We divided the sample into five education groups: high school dropouts (HSD), high school graduates (HSG), individuals with some college (SC), college graduates (CG) and post-college degree holders (PC). We measure education using the variable "educ" constructed by IPUMS. We use the schooling data from 17 to 30 in estimation, and assume no one attends school after age 30. We define unmarried as including separated, widowed, divorced and never married.

Cohort	Observations	Observation per year	Age availability
1960	577,387	5,105	bU
1970	481,395	5,407	
1980	345,496	13,848	
1990	242,959	3,634	

Table A.1: Descriptive statistics

In order to construct couples, we kept only heads of households and spouses (i.e., no secondary families were used), and dropped households with more than one male or more than one female adult. We then merged women and men based on year and household id and dropped problematic couples (with two heads or two spouses, with more than one family or with inconsistent marital status or number of children).

Nominal wages are deflated using the Personal Consumption Expenditure (PCE) index from NIPA Table 2.3.4 [\(http://www.bea.gov/national/nipaweb/index.asp\)](http://www.bea.gov/national/nipaweb/index.asp). Since wages refer to the previous year, we use the PCE for year *t*-1 to deflate observations in year *t*. All wages are expressed in constant 2017 dollars. The top-coded wage observations up until 1995 are multiplied by 1.75.

B.2. Health Data and Health Transition Process

We take the health data from the [IHIS](https://www.ihis.us/ihis/) (integrated health interview series) at Minnesota. The survey contains a subjective health index that takes on 5 values: Excellent, Very Good, Good, Fair, Poor. Empirically, we see that wages are not very different for those in the top three categories, but they are lower for those in fair or poor health. Thus, we decided to merge {Excellent, Very Good, Good} into a single category of "Good" health, and {Fair, Poor} into a single category of "Poor" health. We then calculated the cumulative distribution of this new variable by cohort, gender, ethnicity and age. We assume that each person starts out in good health at age 17, and estimate the transition probability to "Poor" health for each cohort and group. The parameters of the health transition matrix for each cohort are:

		White			Black		Hispanic			
	1960 !980 1970			1960	970	1980	1960	1970	1980	
Good to Good Health	94.4%	93.5%	95.2%	89.6%	88.9%	90.4%	89.4%	89.9%	88.4%	
Poor to Poor Health	85.5%	86.5%	83.4%	89.2%	91.1%	90.0%	89.7%	92.4%	88.7%	

Table B.2: Health transition function parameters

B.3. Social Welfare Payments

Historically, social welfare benefits in the US were heavily targeted toward single women with children, who were often viewed as a "deserving" group.[19](#page-14-0) These benefits include AFDC/TANF, public housing and child care subsidies (for women who work). They also include Medicaid, Foodstamps and other programs. As Keane and Moffitt (1998) discuss, the determination of welfare benefits for single mothers is extremely complicated. This is because of the large number of programs, the fact that participation is a choice (and many women do not take up benefits), and the fact that program benefit rules are both individually complex and interact in complex ways. Indeed, welfare benefits can't be expressed as a simple function of income (or labor supply) and children.^{[20](#page-14-1)}

Given this complexity, we decided to specify the whole array of social benefits targeted at single mothers by a simple exogenous process. Thus, in equation (3) of Section III.A of the main text, we assume single mothers are entitled to social welfare benefits that depend on income and the number of children. We estimate this regression from CPS data, and treat the parameters as exogenously given when estimating our structural model. 21

Specifically, we measure welfare benefits in the CPS using the variable "INCWELFR" (i.e., income from welfare) in IPUMS. This indicates how much pre-tax income (if any) the respondent received during the previous calendar year from various public assistance programs commonly referred to as "welfare." We adjust for inflation using the PCE (just as we did with wages). We then ran a regression of the real annual welfare payment as a function of annual income and the number of children using the subsample of single mothers with children who take up benefits. We run the regression separately until the 1996 welfare reform and after it. We obtain the estimates:

	before 1996	after 1996
constant	4358	3102
<i>n</i> come	-0.079	-0.061
per child	765.	

Table B.3: Welfare payments Parameters as function of children and income

After 1996 we also introduce a 5 year time-limit on welfare. We also allow for the welfare stigma parameter to change before and after the 1996 reform. It increases from 10.7 utility units before 1996 to 279 afterwards. Note that the welfare reform of 1996, hit the 1960 cohort when they were already 36. It hits the 1970 cohort when the are 26, and for the 1980 cohort it already hit when they were 16.

¹⁹ See Katz, Michael B. (1989), "The Undeserving Poor: From the War on Poverty to the War on Welfare," Pantheon Books, New York.

²⁰ Put more formally. Keane and Moffitt (1998) showed that the budget constraints faced by single women with children in the US are both endogenous (due to program participation decisions) and highly complex. To make matters worse, rules differ substantially by State.

²¹ In our model the $cb_t(N_t)$ process captures a key determinant of the threat point for married women with children when considering divorce.

Appendix C: Taxes

Given the gross income associated with any particular wage offer and labor supply choice, we calculate the household's tax liability based on the federal tax rules in effect in the relevant year. This depends on whether it is a single household (subject to the individual tax schedule) or a married couple (subject to the joint tax schedule), as well as the number of dependents. We also account for the earned income tax credit (EITC). To calculate the tax liability of a given household or individual in any particular year, we collected historical data from 1950-2021 from the following sources:

1. Federal income tax rate history [\(https://taxfoundation.org/federal-tax/individual-income](https://taxfoundation.org/federal-tax/individual-income-payroll-taxes)[payroll-taxes\)](https://taxfoundation.org/federal-tax/individual-income-payroll-taxes)

2. Standard deduction history [\(http://www.taxpolicycenter.org/statistics/standard-deduction\)](http://www.taxpolicycenter.org/statistics/standard-deduction)

3. Personal and dependents exemption [\(https://www.irs.gov/publications/p17/ch03.html\)](https://www.irs.gov/publications/p17/ch03.html)

4. Earned income tax credit parameters [\(http://www.taxpolicycenter.org/statistics/eitc](http://www.taxpolicycenter.org/statistics/eitc-parameters)[parameters\)](http://www.taxpolicycenter.org/statistics/eitc-parameters)

Using these parameters, we programmed a "tax calculator," where the inputs are gross income, marital status, number of children and the year, while the output is net income. The program uses the actual tax brackets and marginal tax rates, the full structure of the EITC and the number of deductions and exemptions (that varies by marital status and number of children).^{[22](#page-15-0)} The tax brackets and marginal rates that were used in the model, together with the full historical data on EITC, deductions and exemptions, can be found in the supplement materials.

To simplify the solution of the dynamic programming problem, we assume that agents in the model have perfect foresight about future tax rules. Furthermore, for years beyond 2023, households assume that tax rules will remain fixed at the 2023 values.

There are two possible alternative approaches to modelling expectations: First, we could assume agents are myopic, and every tax rule change is a surprise. This is computationally infeasible, because households would face a new dynamic programming problem each year. Second, we could assume a tax rule generating process, as in Keane and Wolpin (2010). This is also infeasible here, as lagged tax rule parameters become state variables. Perfect foresight is the simplest approach.

Finally, note that we solve the model for three cohorts born in exactly 1960, '70, and '80. When simulating data for hypothetical agents in the model, we use the annual tax rules for persons born in exactly those years. But in the actual data we classify people within a 5-year birth window as part of the same cohort. In estimation we ignore the fact that each data cohort is a mixture of people from five adjacent birth years who face slightly different tax rate histories.

 22 To simplify the program, we assume the maximum number of tax brackets is 10. For some years before 1986, there were more than 10 tax brackets, so we unified similar tax brackets, i.e. instead of having two tax brackets, one with a 49% marginal tax rate and one with 50%, we unified the two into one tax bracket.

Appendix D: Technical Notes on the Solution of the Model

Appendix D.1: The Marriage Market

We assume all married couples are equal in age. We do this for the following reason: Say we back-solve the DP problem from age *T*. Further suppose a person at age *T* may receive marriage offers from either: (i) people who are also age *T*, or (ii) people who are younger. In the case of an offer from a potential partner who is also age *T*, we can easily calculate the expected value of the marriage state at age *T* for both parties. We can then compare this to the expected value of being single. Then, by comparing the married and single value functions, we can determine if the marriage will form. These calculations are straightforward because there is no future $(T+1)$ for either party, so it is a static problem.

On the other hand, suppose a person of age *T* receives a marriage offer from a younger person. To be concrete, say the latter is age *T*-1. Then we run into a major problem: Because we are still in the process of solving for the age *T* value functions, we do not yet have the information we need to calculate age *T*-1 value functions. As a result, we cannot determine the value of the match for the person of age *T*-1. Hence, we cannot determine if the match will form. Given this conundrum, it appears to be impossible to solve a dynamic marriage market model (using the method of back-solving) if people can get offers from younger people.^{[23](#page-16-0)} We avoid this problem by assuming couples are equal in age.

An alternative approach would be to drop chronological age from the state space entirely. For instance, one could replace chronological age by biological age, and assume this is a state variable that evolves stochastically – e.g., biological age could go up, down or stay the same from *t* to *t*+1, depending on what happens to a person's health. We might assume that when a person reaches chronological age *T*+1 they die with certainty. Nevertheless, this would be an infinite horizon problem, because even a person of biological age *T* has a positive survival probability. The solution to such a model would be obtained by solving a fixed-point problem, not by back-solving.

In this type of model, a person of biological age *t* could potentially receive marriage offers from people of any biological age from $t=1,\ldots,T$. This no longer creates a problem, because the model could be solved using a fixed-point method, rather than by back-solving. So, if replace chronological age by biological age in the state space, the fact that a person may receive marriage offers from a younger person creates no (fundamental) computational problem.

We decided not to adopt this approach for two reasons: The first is the computational complexity of solving for a fixed point in a model was complex as ours. Second, if chronological age is not in the state space, it seems difficult to generate the observed similarity of ages within married couples. We could obviously introduce a preference for marrying someone of similar biological age. But the distribution of health, which is the main signal of biological age, is rather stable across different chronological ages in our data, at least until people reach their 60s and 70s. Thus, even a strong tendency to marry people of similar biological age would likely leave us with a counterfactually large dispersion of chronological ages within couples. Better data on markers for biological age could resolve this problem. For now, we decided that an assumption of equal chronological ages within couples would be simpler to implement, and would provide a reasonable approximation to the data, as most couples are fairly close in age.

²³ Note that making the maximum age of marriage less than *T* would not change the nature of this problem.

Appendix D.2: The Number and Ages of Children

The state space becomes extremely large if we follow the ages of children. Therefore, we introduce, in equation (A7), dynamics in the utility of home time following the birth of a new child. If children only entered the model through their effect on tastes for leisure, then neither $N(t)$ not the ages of children would be state variables (as μ_{it} in equation (A7) is a sufficient statistic for all past fertility).

However, *N*(*t*) also enters our model through the budget constraint (the cost of children), welfare rules, tax rules and the cost of divorce. These quantities depend only on *N*(*t*), not on the ages of the children. So the addition of these features requires *N*(*t*) to enter the state space. Nevertheless, these features also require a forward-looking agent to foresee when a child will reach age 19, at which point they leave the household and no longer enter payoffs or constraints. Unfortunately, this requires the agent to keep track of the age of the child. And this would render the state space intractably large.

To deal with this, we adopt the position that (i) changes in tastes for leisure when young children arrive is a first-order problem for female labor supply that is captured by μ_{it} , while (ii) relative ages of older children have only a second order effect on labor supply (e.g., labor supply behavior does not vary much as children age from, say, 8 to 18, a view that prior literature supports). Thus, we choose to ignore child ages in the state space, which is tantamount to assuming that, conditional on tastes for leisure μ_{it} , households with the same $N(t)$ will behave in the same way, *ceteris paribus*, regardless of the children's ages. This means that in solving the DP problem we need only solve over a grid of μ_{it} and $N(t)$ values at each age. Of course, when forward simulating the model, we do keep track of the child ages. Thus, for example, if an only child reaches 19, the household ceases to use the value functions defined for *N*=1, and shifts to the value functions defined for *N*=0. Thus, there is a structural break in household behavior when the children leave home.

Appendix E: Details on the Estimation Method

The estimation method is the Method of Simulated Moments (MSM), as proposed by McFadden (1989) and Pakes and Pollard (1989). The method involves finding the parameter vector φ that minimizes the distance between the actual data and data simulated from our model. Let d_r denote a statistic from the actual data, and let $d_r^s(\varphi)$ be the corresponding statistic calculated in the simulated data, and assume we fit the model to $r = 1, \ldots, R$ statistics. We then construct moments of the form:

(D1)
$$
m_r^s(\varphi) = [d_r - d_r^s(\varphi)]
$$
 for $r = 1,..., R$

The vector of simulated moments is given by $g'(\varphi) = [m_1^s(\varphi), ..., m_R^s(\varphi)]$. We minimize the objective function $G(\varphi) = g'(\varphi)Wg(\varphi)$ with respect to φ , where the weighting matrix W is a diagonal matrix consisting of the inverse of the estimated variance of each moment (from a first step). We minimize $G(\varphi)$ with respect to φ using the Simplex algorithm.

Given the solution of the DP problem at a candidate value of *φ*, we simulate data as follows: First, we set the initial conditions at age 17. All agents start with zero work experience, 11 years of education, good health and unmarried. We draw parent characteristics and the skill for each hypothetical person, using the distributions implied by the data and equation (3). We simulate hypothetical data for 5000 men and 5000 women for each cohort. The only difference between cohorts is the initial conditions, specifically, the distribution of parent education, marital status and whether they were born in the US, the stochastic process for health, and the income tax and welfare benefit rules.

Given the initial conditions, we simulate hypothetical life-cycle histories from age 17 until the terminal period.^{[24](#page-18-0)} In order to simulate forward, we must draw, for each person i in each period *t*, the job offer, a wage shock, a taste for leisure shock, a health realization, a taste for marriage shock (if married), and the realization of a potential partner (if single).^{[25](#page-18-1)} For singles we also draw a taste for school shock. And for single women and married couples we draw tastes for pregnancy. Conditional on these draws, the model generates simulated choices and outcomes for all the observed endogenous variables: education, employment, marital status, children, wages, welfare and health.

In order to form the d_r and $d_r^s(\varphi)$ that enter (D1) we construct, for each cohort, a set of statistics from both the simulated and actual data that summarize key predictions of the model. These include (a) the schooling distribution by gender, (b) employment rates by gender, marital status and age, (c) average wages conditional on gender, education, marital status and age, (d) marriage and divorce rates by age, (e) number of children by age/marital status, and (f) the pattern of assortative mating by education of the partners. (g) welfare participation rate of single mothers by employment status and age. We list the moments in detail in Appendix F.

We compute standard errors numerically. Calculation of standard errors is complicated by non-smoothness of the objective function, so we use "long baseline" numerical derivatives, which was one suggestion in McFadden (1989) .^{[26](#page-18-2)} Specifically, we compute the numerical

²⁴ We fit our model only to data up through age 61 to avoid having to model the possible early receipt of Social Security at age 62.

²⁵ We first draw whether a marriage offer is received. If it is, we then draw the match quality and characteristics of the potential partner: schooling, experience, ability, tastes for work, children from previous relationships, health, parents' education and whether employed in the previous period.

²⁶ We admit that long baseline derivatives can be coarse approximations. Thus, Keane (1994) argued for use of smooth simulators (like GHK) instead. Unfortunately, smoothing is difficult given the complexity of our model. Given the structural parameters are of secondary interest here (which is why they are relegated to Appendix G), the standard errors are obviously of secondary interest as well, so we did not view smoothing as essential.

derivative with respect to each of the parameters, φ_p using the five-point stencil formula with a long baseline:^{[27](#page-19-0)}

$$
f_{\varphi_p} = \frac{-f(\varphi_p + 2\varepsilon_p) + 8f(\varphi_p + \varepsilon_p) - 8f(\varphi_p - \varepsilon_p) + f(\varphi_p - 2\varepsilon_p)}{12\varepsilon_p}
$$

Where *f* is a vector of the squared moments divided by their weights: $\left[d_r - d_r^s(\varphi) \right]^2/W_r$, and ε_p is equal to 0.01⋅ φ_p (a rather large gap). Note that the use of baseline intervals of different length is a form of Richardson extrapolation, which is in turn a bootstrapping method. Given the numerical derivatives, we compute the covariance matrix using the outer product approximation to the Hessian.

²⁷ See Abramowitz, Milton; Stegun, Irene A. (1970), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, Dover, 9th Edition. Table 25.2.*

Appendix F: Terminal Value Function Specification

We set a "terminal value function" V_{T+1}^J at a pre-specified age *T* beyond which we do not attempt to structurally model behavior, as in Keane and Wolpin (2001). Because the "normal" Social Security retirement age is 65, and a large fraction of workers do retire by that age, we decided to fix the terminal period *T* at age 65. By setting T=65 we seek to avoid the complications of modeling Social Security and the accumulation of retirement savings. Given the already great complexity of our model, including the additional state and choice variables required to model Social Security benefits and retirement behavior would be infeasible.^{[28](#page-20-0)} Of course, people can choose to stop working earlier that at age 65 if desired.^{[29](#page-20-1), [30](#page-20-2)}

To reduce computational burden, we assume the terminal value function $V_{T+1}^J(\Omega_{j,T+1})$ at $T=65$ is a simple function of the state variables in $\Omega_{i,T+1}$. Specifically, the terminal value function is a linear function of the state variables (dated at the end of period *T*=65) listed in Table F.1. It includes state variables of the spouse if one is married. Table F.1 presents the estimates of the terminal value function parameters, which were obtained separately for men and women:

Table F.1: Terminal Value Function parameters

* Indicates that a parameter is significant at the 5% level.+ indicates that a parameter is significant at the 10% level.

An interesting feature of the terminal value function is that work experience and college education (both own and spousal) are highly significant. In our model these variables play no role other than to determine the distribution of offer wages. Thus, as agents cannot work past age 65, there is no direct reason for them to value these quantities. The only rationale for why

²⁸ For a recent paper that models Social Security and retirement behavior in detail, while also considering labor supply and human capital accumulation, see Keane, M. and N. Wasi (2016), "Labour Supply: the Roles of Human Capital and the Extensive Margin," *The Economic Journal*, 126(592), 578-617. That paper considers only men and does not incorporate marriage or fertility decisions.

²⁹ The "normal" Social Security retirement age is that age at which one can begin receiving full benefits. In fact, the "normal" age for claiming Social Security (SS) benefits was gradually increased from 65 to 67. We ignore this change because age 65 has remained a "focal point" for retirement decisions in the US for two reasons: (i) it is the age for Medicare eligibility, and (ii) it is still a common age for private pension eligibility.

³⁰ It is worth noting that for all our cohorts' workers could also opt to receive "early" Social Security benefits at age 62, subject to a penalty in the form of a reduced benefit level. We ignore this option in our model (that is, we ignore any change in the structure of the model at age 62 that arises because this option is available). To avoid any bias this might cause, we only use data up through age 61 in estimating the model.

agents in our model would care about work experience and education after age 65 is that they proxy for retirement assets. Thus, as we argue in Sections III.F of the main text, the terminal value function accounts for agents' concern about retirement savings in a reduced form way.

Stated another way, the fact that V_{65} is increasing in work experience adds an extra return to labor supply that is not captured by the wage rate wage alone. Presumably, this value arises because labor supply also causes workers to accumulate both Social Security benefits and private pension benefits. If the terminal value function did not incorporate this added value of work, then labor supply in our model would drop off (too) precipitously before the *T*=65 terminal period (as older workers would not need to be concerned about accumulating retirement savings).

Not surprisingly, the terminal value function estimates indicate that people also value marriage and children after age 65. The latter implies that children continue to generate utility for parents even after they leave the household at age 18.

Appendix G: Moments Fit in the MSM Estimation

The moments we can use in estimation depend on the availability of data for each cohort. As we report in Table G.1, we observe different cohorts over different age ranges. Notice that for the 1960, 1970 and 1980 cohorts the data ends at ages 63, 53 and 43, respectively.

Table G.2 lists the set of moments available for each cohort. Since the data for Blacks and Hispanics tend to fluctuate due to relatively low number of observation, we calculated the moments using an average of 5 year intervals: age 22-26, 27-31, 32-36, 37-41, 42-46, 47-51, 52-56, 57-61. We adjusted the number of the moments to the available data. Since the data for ages 17-21 fluctuates substantially, we used data only from age 22 onward. There are 384, 304 and 224 moments for each of the three ethnic groups in the 1960, 1970 and 1980 cohorts, respectively, giving 2736 total.

Table G.2: Data Moments used in Estimation

* 1960 – 41 periods, 8 moments of 5 periods 22-26, 27-31,32-36,…

** 1970 – 33 periods, 6 moments of 5 periods 22-26, 27-31,32-36,…

*** $1980 - 23$ periods, 4 moments of 5 periods 22-26, 27-31, 32-36, 37-41.

**** Schooling distribution from age 17 to 31, no schooling after $31 - 3$ moments: 18-22, 23-27, 28-30 ***** Kids' moments from age 21 to 41. No newborn after 41 – 3 moments: 25-29,30-34, 35-39

****** Welfare moments from age 22 to 41 – 4 moments: 22-26, 27-31, 32-36, 37-41

******* Different source of data: IHIS (integrated health interview series) at Minnesota

+ untargeted moments

Appendix H: Parameter Estimates Table H.1. Utility and Preferences Parameters (Fixed by cohort and ethnic group)

** Indicates that a parameter is significant at the 5% level. * indicates that a parameter is significant at the 10% level.

A " indicates that men and women share the same parameter value.

Table H.2 reports the estimates of the offer wage function, equation (2) in the main text:

Table H.2. Wage Function Parameters (estimated separately for each cohort and group)

* Indicates that a parameter is significant at the 5% level.

Table H.3 reports the estimates of the job offer and job destruction probability functions in equations (5) and (4). Recall that (5) determines the probabilities that unemployed workers receive part- and or full-time offers. Workers who were employed in the previous period are assumed to be able to continue working, unless there is a forced separation.

The bottom panel of Table H.3 reports the parameters of the logit model for exogenous job separations, which is equation (4) in main the text. The outcome is defined as 1 if a worker can keep their job and 0 if an exogenous separation occurs. The estimates imply that experience and education reduce the separation probability, while poor health increases it.

* Indicates that a parameter is significant at the 5% level.

Finally, Table H.4 reports estimates of the marriage market matching process in Appendix A, equation (A23):

Table H.4. Marriage Market Parameters (Estimated separately for each cohort and group)

Appendix I: Assortative Mating Patterns for Blacks and Hispanics

Assortative Mating Patterns by Cohort, Black only

Assortative Mating Patterns by Cohort, Hispanic only

Appendix J: Additional Model Fit Results

Table J1 shows the model fit to wages, employment and welfare participation. Fit is very good on all three dimensions. For example, the fraction of single mothers who receive welfare benefits declined dramatically across the three cohorts, and our model captures this well. For Whites in the 1960, '70 and '80 cohorts, the fraction of unemployed single mothers aged 27-31 who participated in welfare fell from 60% to 33% to 13%, and our model predicts a decline from 56% to 35% to 10%. For Blacks and Hispanics welfare participation rates are higher but also fell substantially. For instance, for Blacks in the 1960 cohort, 71% of unemployed single mothers aged 27-31 participated in welfare, but in the 1980 cohort this dropped to only 24%. Our model predicts a decline from 75% to 20%.

Finally, Table J2 shows the fit to marriage and divorce rates, total fertility at age 35, and educational assortative mating. Marriage rates fall across cohorts for all three ethnic groups, especially at young ages (27-31). The model captures this pattern well. Total fertility at age 35 is fairly stable for all groups, and the model also captures this. The rate of HSG women matching with HSG men falls, while rates of CG women matching with CG men, and PC women matching with PC men, increase across cohorts, for all groups, and the model captures this well.

	1960			1970		1980 1960		1970		1980		1960		1970		1980		
	White		White			White		Black		Black	Black		Hispanic		Hispanic		Hispanic	
																	ActualFitted Actual Fitted	
Family moments by age group																		
Marriage Rate - 27-31 0.68 0.66 0.63 0.65 0.58 0.61 0.37 0.40 0.32 0.35 0.28 0.31 0.65 0.62 0.62																	0.64 0.57 0.55	
Marriage Rate - 32-36 0.73 0.72 0.70 0.69 0.67 0.69 0.40 0.43 0.39 0.39 0.34 0.36 0.65 0.65 0.67																$0.69 \, \, 0.61$		0.62
Marriage Rate - 37-41				0.73 0.72 0.71 0.71	$\vert 0.69 \vert 0.70 \vert 0.41 \vert 0.43 \vert$										0.40 0.40 0.38 0.39 0.66 0.67 0.65		$0.69 \mid 0.63 \mid 0.64$	
Divorce Rate - 27-31			$0.10 \mid 0.12 \mid 0.08 \mid 0.09$												$\vert 0.07 \vert 0.08 \vert 0.08 \vert 0.09 \vert 0.07 \vert 0.09 \vert 0.05 \vert 0.03 \vert 0.08 \vert 0.09 \vert 0.05 \vert$	0.07	0.05	0.07
Divorce Rate - 32-36					0.11 0.13 0.11 0.10 0.10 0.10 0.14 0.13 0.11 0.10 0.09 0.10 0.10 0.09 0.08												$0.10 \mid 0.08 \mid 0.09$	
Divorce Rate - 37-41				0.14 0.13 0.14 0.13	0.13 0.11		0.17				$\vert 0.15 \vert 0.15 \vert 0.13 \vert 0.13 \vert 0.13 \vert 0.13 \vert 0.11 \vert 0.11$						0.10 0.10 0.09	
Married Women - kids distribution at age 35																		
Childlessness rate				0.14 0.15 0.17 0.16	$\vert 0.17 \vert 0.19 \vert 0.13 \vert 0.13 \vert 0.17 \vert 0.14 \vert 0.19 \vert 0.18 \vert 0.13 \vert 0.10 \vert 0.11$											$0.13 \, 10.11$		0.13
Number of Children		1.89 1.94 1.77		1.70		1.84 1.71		1.99 2.12			$1.96 \mid 1.98 \mid 1.92 \mid 1.91$				2.25 2.33 2.22		2.20 2.24 2.15	
Un-Married Women - kids distribution at age 35																		
Childlessness rate				0.50 0.52 0.53 0.53													$\vert 0.51 \vert 0.53 \vert 0.23 \vert 0.23 \vert 0.32 \vert 0.31 \vert 0.35 \vert 0.34 \vert 0.31 \vert 0.27 \vert 0.30 \vert 0.32 \vert 0.31 \vert 0.32$	
Number of Children			0.92 0.99 0.89 0.91				$0.93 \mid 0.81 \mid 1.73 \mid$										$\vert 1.74 \vert 1.54 \vert 1.61 \vert 1.51 \vert 1.43 \vert 1.60 \vert 1.61 \vert 1.58 \vert 1.55 \vert 1.62 \vert 1.50$	
Assortative Mating																		
HSD with HSD					$0.36 \mid 0.38 \mid 0.27 \mid 0.29 \mid 0.31 \mid 0.34 \mid 0.38 \mid 0.42 \mid 0.32 \mid 0.36 \mid 0.33 \mid 0.35 \mid 0.74 \mid 0.65 \mid 0.73$												$0.66 \mid 0.69 \mid 0.64$	
HSG with HSG				0.6010.6310.4910.52													$(0.40 \mid 0.43 \mid 0.53 \mid 0.56 \mid 0.49 \mid 0.51 \mid 0.44 \mid 0.46 \mid 0.53 \mid 0.57 \mid 0.52 \mid 0.56 \mid 0.51 \mid 0.54$	
SC with SC			0.46 0.48 0.48 0.53		$0.46 \mid 0.49 \mid$										$\vert 0.53 \vert 0.46 \vert 0.52 \vert 0.53 \vert 0.49 \vert 0.53 \vert 0.45 \vert 0.48 \vert 0.46 \vert$		$0.49 \, 0.48$	0.49
CG with CG		$0.46 \mid 0.51$	0.51	0.52			$0.51 \mid 0.54 \mid 0.40 \mid$	0.43							0.44 0.48 0.42 0.45 0.40 0.43 0.43		$0.44 \, \, 0.43$	0.45
PC with PC																	$0.31\, 0.34\, 0.41\, 0.46\, 0.49\, 0.51\, 0.38\, 0.40\, 0.40\, 0.43\, 0.47\, 0.51\, 0.25\, 0.31\, 0.34\, 0.39\, 0.48\, 0.50\, 0.46\, 0.50\, 0.47\, 0.51\, 0.51\, 0.55\, 0.56\, 0.57\, 0.58\, 0.59\, 0.59\, 0.59\, 0.50\, 0.5$	

Table J2: Model Fit to Family Moments and Educational Assortative Mating

Appendix K: Job Offer and Destruction Rates by Work Experience

Table K1 shows how job offer rates vary with the level of work experience. This supplements Table 7 that reports results at 3 years of experience.

	Women 1960		Men 1960		women 1980		Men 1980		
	HSG	CG	HSG	CG	HSG	CG	HSG	CG	
White									
$EXP = 3$	0.41	0.46	0.52	0.59	0.48	0.55	0.51	0.60	
$EXP = 5$	0.44	0.49	0.56	0.63	0.50	0.58	0.55	0.64	
$EXP = 10$	0.51	0.56	0.66	0.72	0.57	0.64	0.65	0.73	
Black									
$EXP = 3$	0.30	0.31	0.34	0.36	0.35	0.37	0.39	0.41	
$EXP. = 5$	0.31	0.32	0.35	0.37	0.36	0.39	0.41	0.43	
$EXP = 10$	0.34	0.35	0.38	0.40	0.40	0.42	0.46	0.48	
Hispanics									
$EXP = 3$	0.36	0.41	0.48	0.53	0.39	0.46	0.51	0.58	
$EXP = 5$	0.38	0.43	0.52	0.57	0.41	0.49	0.55	0.62	
$EXP = 10$	0.44	0.50	0.60	0.65	0.49	0.56	0.66	0.72	

Table K1: Job offer rate by race and cohort, by experience and education

Table K2 reports selected job destruction rates. This supplements Table 8 that reports rates at 3 years of experience.

Table K2: Job destruction rate by race and cohort, by experience and education

	Women 1960		Men 1960		women 1960		Men 1960		
	HSG	CG	HSG	CG	HSG	CG	HSG	CG	
White									
$EXP = 3$	0.15	0.12	0.12	0.10	0.14	0.11	0.14	0.12	
$EXP = 5$	0.11	0.09	0.07	0.06	0.10	0.08	0.09	0.08	
$EXP = 10$	0.04	0.03	0.02	0.02	0.03	0.03	0.03	0.03	
Black									
$EXP = 3$	0.25	0.23	0.27	0.25	0.24	0.22	0.25	0.23	
$EXP = 5$	0.22	0.21	0.25	0.23	0.21	0.19	0.22	0.20	
$EXP = 10$	0.16	0.15	0.19	0.18	0.15	0.14	0.15	0.14	
Hispanic									
$EXP = 3$	0.17	0.16	0.15	0.13	0.17	0.15	0.15	0.14	
$EXP = 5$	0.13	0.12	0.11	0.09	0.13	0.11	0.11	0.10	
$EXP = 10$	0.07	0.07	0.04	0.04	0.06	0.06	0.04	0.04	